M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
II Year IV Semester
Core Major- Paper XI
DIFFERENTIAL GEOMETRY AND TENSOR CALCULUS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Write down the Serret-Frenet formula for a space curve.
2. State the fundamental existence theorem for space curves.
3. Define an anchor ring.
4. Define an ordinary point for a space curve.
5. State the normal property of geodesics.
6. State Whitehead existence theorem.
7. Define covariant tensor of order two.
8. Show that the Kronecker delta δ_i^j is a mixed tensor of order two.
9. Prove that $[\, ij,\ k] + \ [kj,\ i] = \frac{\partial g_{ik}}{\partial x^j}$.
10. Define Christoffel symbols of first kind.
11. Define cylindrical helix.
12. Define symmetric tensor.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that $[\ \dot{r}, \ddot{r}, \dddot{r}\ ] = 0$ is a necessary and sufficient condition that the curve be planar.
14. Find the orthogonal trajectories of the sections by the planes $z =$ constant , on the paraboloid $x^2 - y^2 = z$.
15. Prove that the curve of the family $\frac{v^3}{u^2} = \text{constant}$ are geodesics on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2 \ (u > 0,\ v > 0)$.
16. State and prove Liouville's formula for κ_g.
17. Show that a skew symmetric tensor of the second order has only $\frac{n\,(n-1)}{2}$ different non-zero component.

18. If g denotes the determinant $|g_{ij}|$ prove that $\frac{\partial}{\partial x^i}\log\sqrt{g}=\left\{ \begin{matrix} \alpha \\ i\alpha \end{matrix} \right\}$.

19. Show that the Kronecker delta is constant with respect to covariant differentiation.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2+by^2+cz^2=1,\ a'x^2+b'y^2+c'z^2=1$.

21. Define the metric on a given surface $\vec{r}\,(u,v)$. Show that the metric is positive definite and remains invariant under the transformation $u'=\emptyset\,(u,v)$ and $v'=\psi\,(u,v)$.

22. State and prove Gauss-Bonnet theorem.

23 Prove that the sum(or difference) of two tensors which have the same rank of covariant and the same number of contravariant indices is again a tensor of the same type and rank as the given tensor.

24. State and prove Ricci's theorem.

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