M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
II Year IV Semester
Core Major
FUNCTIONAL ANALYSIS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define uniform norm.
2. Define bounded linear transformation.
3. Define isomorphism of two normed linear spaces.
4. State uniform boundedness theorem.
5. Define an orthonormal set.
6. Prove that $(\mathbf{T_1 T_2})^{*}=\mathbf{T_2^{*} T_1^{*}}$.
7. Define normal operator.
8. Define an unitary operator.
9. Define Banach- Algebra.
10. Define spectrum of x.
11. Define Banach*-Algebra.
12. Define multiplicative functional.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that $l_{\mathbf{p}}^{\mathbf{n}}$ is a Banach space.
14. State and prove Minkowski's inequality.
15. Let M be a closed linear subspace of a Hilbert space H, let x be a vector not in M and let d be the distance from x to M. Prove that there exists a unique vector $\mathbf{y_0}$ in M such that $||\mathbf{x}-\mathbf{y_0}|| = \mathbf{d}$.
16. State and prove Bessel's inequality.
17. If $\mathbf{N_1}$ and $\mathbf{N_2}$ are normal operators on H with the property that either commutes with the adjoint of the other, then prove that $\mathbf{N_1}+\mathbf{N_2}$ and $\mathbf{N_1 N_2}$ are normal.
18. If 0 is the only topological divisor of zero in A then prove that $\mathbf{A} = \mathbf{C}$.
19. If A is self adjoint, prove that $\hat{\mathbf{A}}$ is dense in $\mathcal{C}(\mathfrak{M})$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x+M in the quotient space N/M is defined by $||\mathbf{x+M}|| = \mathbf{i}nf\{||\mathbf{x+m}|| : \mathbf{m} \in \mathbf{M}\}$ then prove that N/M is a normed linear space. Further, if N is a Banach space, then prove that N/M is also a Banach space.
21. State and prove closed graph theorem.
22. Let H be a Hilbert and let f be an arbitrary functional in H*. Prove that there exists a unique vector y in H such that $\mathbf{f(x)} = <\mathbf{x}, \mathbf{y}>$ for all x in H.
23. Prove that $\mathbf{r}\,(\mathbf{x}) = \mathbf{l}im||\mathbf{x^n}||^{\mathbf{1/n}}$.
24. State and prove Gelfand-Newmark representation theorem.

M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
II Year IV Semester
Core Major
FUNCTIONAL ANALYSIS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define uniform norm.
2. Define bounded linear transformation.
3. Define isomorphism of two normed linear spaces.
4. State uniform boundedness theorem.
5. Define an orthonormal set.
6. Prove that $(\mathbf{T_1T_2})^{*}=\mathbf{T_2^{*}T_1^{*}}$.
7. Define normal operator.
8. Define an unitary operator.
9. Define Banach- Algebra.
10. Define spectrum of x.
11. Define Banach*-Algebra.
12. Define multiplicative functional.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that $l_{\mathbf{p}}^{\mathbf{n}}$ is a Banach space.
14. State and prove Minkowski's inequality.
15. Let M be a closed linear subspace of a Hilbert space H, let x be a vector not in M and let d be the distance from x to M. Prove that there exists a unique vector $\mathbf{y_0}$ in M such that $||\mathbf{x}-\mathbf{y_0}||=\mathbf{d}$.
16. State and prove Bessel's inequality.
17. If $\mathbf{N_1}$ and $\mathbf{N_2}$ are normal operators on H with the property that either commutes with the adjoint of the other, then prove that $\mathbf{N_1}+\mathbf{N_2}$ and $\mathbf{N_1N_2}$ are normal.
18. If 0 is the only topological divisor of zero in A then prove that $\mathbf{A} = \mathbf{C}$.
19. If A is self adjoint, prove that $\hat{\mathbf{A}}$ is dense in $\mathcal{C}\left(\mathfrak{M}\right)$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x+M in the quotient space N/M is defined by $||\mathbf{x+M}|| = \mathbf{i}nf\{||\mathbf{x+m}|| : \mathbf{m} \in \mathbf{M}\}$ then prove that N/M is a normed linear space. Further, if N is a Banach space, then prove that N/M is also a Banach space.
21. State and prove closed graph theorem.
22. Let H be a Hilbert and let f be an arbitrary functional in H*. Prove that there exists a unique vector y in H such that $\mathbf{f}(\mathbf{x}) = <\mathbf{x}, \mathbf{y}>$ for all x in H.
23. Prove that $\mathbf{r}\,(\mathbf{x}) = \mathbf{l}im||\mathbf{x}^{\mathbf{n}}||^{\mathbf{1/n}}$.
24. State and prove Gelfand-Newmark representation theorem.