M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
I Year I Semester
Core Major -II
REAL ANALYSIS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Lebesque outer measure for subsets of real numbers.
2. Define σ- algebra.
3. State Fatou's Lemma.
4. State Lebesque's Monotone Convergence theorem.
5. Define Uniform Convergence.
6. State Weierstrass M-test.
7. Define Contraction mapping of a Metric space.
8. State inverse function theorem.
9. Define orthogonal system of function.
10. State Stirling's formula.
11. Show that if F is measurable and $\mathbf{m}^*(\mathbf{F}\triangle\mathbf{G}) = \mathbf{0}$, then G is measurable
12. Let $\mathbf{f}$ and $\mathbf{g}$ be integrable functions then prove that $\mathbf{a}f$ is integrable, and $\int \mathbf{a}f\mathbf{d}x = \mathbf{a}\int \mathbf{f}dx.$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that every interval is measurable.
14. Show that if $\alpha > \mathbf{1}$, $\int_{\mathbf{0}}^{\mathbf{1}} \frac{x\mathbf{s}inx}{\mathbf{1}+(\mathbf{n}x)^{\alpha}}\mathbf{d}x = 0(n^{-1})$ as $n \to \infty$.
15. State and prove theorem on Cauchy condition for uniform convergence of sequences.
16. Suppose f maps a convex open set $\mathbf{E} \subset \mathbf{R}^{\mathbf{n}}$ $\mathbf{i}nto$ $\mathbf{R}^{\mathbf{m}}$, $\mathbf{f}$ is differentiable in E, and there is a real number M such that $\left\|\mathbf{f}'(\mathbf{x})\right\| \leq \mathbf{M}$ $\mathbf{f}or$ $\mathbf{e}very$ $\mathbf{x} \in \mathbf{E}$. Then Prove that $|\mathbf{f}(\mathbf{b}) - \mathbf{f}(\mathbf{a})| \leq \mathbf{M}|\mathbf{b} - \mathbf{a}|$ $\mathbf{f}or$ $\mathbf{a}ll$ $\mathbf{a} \in \mathbf{E}, \mathbf{b} \in \mathbf{E}$.

17. Suppose $\sum \mathbf{c_n}$ converges, let $\mathbf{f}(\mathbf{x}) = \sum_{\mathbf{n=0}}^{\infty} \mathbf{c_n x^n} \quad (\mathbf{-1<x<1})$ then Prove that $\lim_{\mathbf{x\to 1}} \mathbf{f}(\mathbf{x}) = \sum_{\mathbf{n=0}}^{\infty} \mathbf{c_n x^n}$
18. State and prove Lebesque's Dominated Convergence theorem.
19. Prove that not every measurable set is a Borel set.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. (a) Let $\{\mathbf{E_i}\}$ be a sequence of measurable sets . Then Prove that
    i. If $\mathbf{E_1} \subseteq \mathbf{E_2} \subseteq \ldots,$ we have $\mathbf{m}(\mathbf{E_i}) = \mathbf{l}im\ \mathbf{m}(\mathbf{E_i})$
    ii. If $\mathbf{E_1} \supseteq \mathbf{E_2} \supseteq \ldots,$ and $\mathbf{m}(\mathbf{E_i}) < \infty$ $\mathbf{f}or\ \mathbf{e}ach\ \mathbf{i}$, then we have $\mathbf{m}(\mathbf{l}imE_\mathbf{i}) = \mathbf{l}im\ \mathbf{m}(\mathbf{E_i})$

    (b) For any sequence of sets $\{\mathbf{E_i}\}$, Show that $\mathbf{m}^*\left(\bigcup_{\mathbf{i=1}}^{\infty} \mathbf{E_i}\right) \leq \sum_{\mathbf{i=1}}^{\infty} \mathbf{m}^*(\mathbf{E_i})$.
21. If $\mathbf{f}$ is Riemann integrable and bounded over the finite interval [a,b] then Prove that $\mathbf{f}$ is integrable and R $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{f}dx = \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{f}dx$
22. State and prove the Stone – Weierstrass theorem.
23. State and prove the Implicit function theorem.
24. State and prove Bessel's inequality.

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    ii. If $\mathbf{E_1} \supseteq \mathbf{E_2} \supseteq \ldots,$ and $\mathbf{m}\left(\mathbf{E_i}\right)<\infty$ $\mathbf{f}or\ \mathbf{e}ach\ \mathbf{i},$ then we have $\mathbf{m}\left(\lim E_\mathbf{i}\right)=\lim\ \mathbf{m}\left(\mathbf{E_i}\right)$

    (b) For any sequence of sets $\{\mathbf{E_i}\}$, Show that $\mathbf{m}^*\left(\bigcup_{\mathbf{i=1}}^{\infty}\mathbf{E_i}\right) \leq \sum_{\mathbf{i=1}}^{\infty}\mathbf{m}^*\left(\mathbf{E_i}\right).$
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