

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2018**  
**I Year II Semester**  
**Core Major - Paper V**  
**TOPOLOGY**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. Define a complete metric space and give an example.
2. Define a metrizable space.
3. Define product topology on  $X$ .
4. Define a compact space.
5. Define a totally bounded set and give an example.
6. Define the term "equicontinuous" for functions on a compact metric space.
7. Define a completely regular space.
8. Define a normal space.
9. Define a totally disconnected space and give an example.
10. Define components of a topological space and illustrate with an example.
11. Define an open base for a metric space and give an example.
12. Define a sequentially compact metric space.

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. Prove that a closed subspace of a complete metric space is complete.
14. If  $X$  is a second countable space, then show that any open base for  $X$  has a countable subclass which is also an open base.
15. Show that every sequentially compact metric space is totally bounded.
16. Prove that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism.
17. Prove that any continuous image of a connected space is connected.
18. Show that the product of any non-empty class of Hausdorff space is a Hausdorff space.
19. If  $f$  and  $g$  are continuous real functions on a metric space  $X$ , then prove that  $f + g$  is also continuous.

**Section C** ( $3 \times 10 = 30$ ) Marks

Answer any **THREE** questions

20. If  $X$  and  $Y$  are metric spaces and  $f$  a mapping of  $X$  into  $Y$ , prove that  $f$  is continuous at  $x_0$  if and only if  $x_n \rightarrow x_0$  implies  $f(x_n) \rightarrow f(x_0)$ .
21. State and prove Heine Borel theorem.
22. State and prove Tychonoff 's theorem.
23. State and prove Urysohn's lemma.
24. If  $X$  is a compact Hausdorff space, then show that  $X$  is totally disconnected if and only if it has an open base whose sets are also closed.

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