M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
II Year III Semester
Core Major -VII
COMPLEX ANALYSIS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define an entire function and give an example.
2. Give an example of a region which is simply connected and a region which is not simply connected.
3. Define singular part of an analytic function f(z).
4. Determine the singularity and type for the function $\frac{\sin z}{z}$.
5. Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.
6. Prove that $\lim_{z\to 0} \frac{\log(1+z)}{z} = 1$
7. Define Poisson kernel.
8. Define a Dirichlet region and give an example.
9. Define finite order and infinite order of an entire function.
10. State Schottky's theorem.
11. State Hadamard's factorization theorem.
12. Define subharmonic and superharmonic functions.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. State and prove Liouville's theorem.
14. State and prove Rouche's theorem.
15. If Re $z_n > -1$, then prove that the series $\sum \log(1+z_n)$converges if and only if the series $\sum z_n$ converges absolutely.
16. State and prove Harnack's inequality.
17. Suppose g is an analytic function on B (0; R), g (0) =0,$|g'(0)| = \mu > 0$ and $|g(z)| \leq M$,for all z, then show that $g(B(0;R)) \supset B(0; \frac{R^2\mu^2}{6M})$.

18. If f: D$\rightarrow$D is an one - one analytic map of D onto itself and f(a) $=$0 ,then prove that there is a complex number c with $|c| = 1$ such that f $=$cϕ_α.

19. If Re z $>$ 1, show that $\zeta(z) = \prod_{n=1}^{\infty}\left(\frac{1}{1-p_n^{-z}}\right)$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove Goursat's theorem on analytic functions.

21. State and prove Residue theorem.

22. State and prove Riemann mapping theorem.

23. Let G be a region and let a $\in \partial_\infty G$ such that there is a barrier for G at a.
If f: $\partial_\infty G \rightarrow \mathbb{R}$ is continuous and u is the Perron function associated with f, then show that $\lim_{z\to a} u(z) = f(a)$.

24. State and prove Bloch's theorem.

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