M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
I Year I Semester
Core Major -I
ALGEBRA - I

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Write down the class equation of group *G*.
2. State Cauchy's Theorem.
3. Define internal direct product of normal subgroups of group *G*.
4. Define R-module.
5. Define trace of a matrix.
6. Check whether $\begin{pmatrix} 1 & 0 & i \\ 0 & 2 & 3+i \\ -i & 3-i & -1 \end{pmatrix}$ is Hermitian or not.
7. Define finite field and give an example.
8. Prove that any two finite fields having the same number of elements are isomorphic.
9. When is a division algebra said to be algebraic over a field.?
10. For all $x, y \in Q$, Prove that $N(xy) = N(x)N(y)$.
11. Define normaliser of *a* in *G*
12. When is a group said to be solvable.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If *G* is a finite group with $o(G) = p^n$ where p is a prime number, Prove that $Z(G) \neq \{e\}$.
14. Prove that S_n is not solvable for $n \geq 5$.
15. Prove that a linear transformation *T* on *V* is unitary if and only if it takes an orthonormal basis of *V* into an orthonormal basis of *V*.
16. If *F* is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of *F*, Prove that we can find elements $a, b \in F$ such that $1 + \alpha a^2 + \beta b^2 = 0$.
17. State and prove the four square theorem.
18. If *A* and *B* are finite subgroups of group *G*, Prove that $o(AxB) = \dfrac{o(A)o(B)}{o(A \cap Bx^{-1})}$

19. If *G* is a group, *G* is the internal direct product of $N_1, N_2, N_3, ...N_n$ and $T = N_1 \times N_2 \times N_3 \times ... \times N_n$,Prove that *G* and *T* are isomorphic.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove Sylow's theorem.
21. Prove that every finite abelian group is the direct product of cyclic groups.
22. If *F* is a field of characteristic 0 and if $T \in A(V)$ is such that $T^i = 0$ for all $i \geq 1$, Prove that *T* is nilpotent.
23. State and prove Wedderburn's theorem.
24. State and prove Frobenius theorem.

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