M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
I Year II Semester
Core Major-Paper IV
ALGEBRA - II

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define extension field and give an extension of the field of real numbers.
2. When do you say that an element $a \in K$ is algebraic of degree *n* over *F*?
3. Define algebraic and transcendental numbers.
4. State the remainder theorem.
5. For any $f(x), g(x)$ in *F*[*x*] and $\alpha \in F$, Show that $(\alpha f(x))^1 = \alpha f^1(x)$.
6. Define field automorphism and when do you say that two automorphisms of a field are distinct.
7. Define $G(K, F)$.
8. When do you say linear transformations $S, T \in A(V)$ are similar.
9. Define index of nilpotence of $T \in A(V)$.
10. Write down the Jordan form of a linear transformation.
11. Define normal extension of a field.
12. Define invariant subspace of a vector space.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If *L* is an algebraic extension of *K* and if *K* is an algebraic extension of *F*, Prove that *L* is an algebraic extension of *F*.
14. Prove that a polynomial of degree *n* over a field can have at most *n* roots in any extension field.
15. Show that $\sqrt{2} + \sqrt[3]{5}$ is algebraic over *Q* of degree.
16. If *V* is *n*-dimentional over *F* and $T \in A(V)$ if has all its characteristics roots in *F*, Prove that *T* satisifies a polynomial of degree *n* over *F*.

17. If $T \in A(V)$has all its distinct characteristic roots $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ in *F*, Prove that a basis of *V* can be found in which the matrix of *T* is of the form $\begin{pmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_k \end{pmatrix}$
18. Prove that the polynomial $f(x) \in F[x]$has a multiple root if and only if $f(x)$ and $f^1(x)$have a nontrivial common factor.
19. Prove that *K* is a normal extension of *F* if and only if *K* is the splitting field of some polynomial over *F*.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the element $a \in K$ is algebraic over *F*, if and only if *F*(*a*) is a finite extension of *F*.
21. If *F* is of characteristic 0 and if *a*, *b* are algebraic over *F*, Prove that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.
22. If *K* is a finite extension of *F*, Prove that $G(K, F)$ is a finite group and its order $o(G(K, F))$ satisfies $o(G(K, F)) \leq [K : F]$.
23. If $T \in A(V)$ has all its characteristic roots in *F*, Prove that there is a basis of *V* in which the matrix of *T* is triangular.
24. Prove that the elements S and T in $A_F(V)$are similar in $A_F(V)$ if and only if they have the same elementary divisors.

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