M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
II Year III Semester
Core Major -VIII
DIFFERENTIAL EQUATIONS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define analytic function.
2. Check whether $\mathbf{t{=}0}$ is an ordinary point for the Hermite equation.
3. Define a Fundamental matrix
4. Find the fundamental matrix for the system $x' = Ax$ where $\mathbf{A}= \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$
5. Show that $\mathbf{f}\left(\mathbf{t},\mathbf{x}\right)=\mathbf{x}^{\frac{1}{2}}$ be defined on the rectangle $\mathbf{R}=\{\left(\mathbf{t},\mathbf{x}\right): |\mathbf{t}| \leq \mathbf{1}, \quad |\mathbf{x}| \leq \mathbf{2}$ do not satisfy Lipschitz condition.
6. State Gronwall Inequality.
7. Eliminate $\mathbf{a}$ and $\mathbf{b}$ from $\mathbf{z}=\mathbf{a}x\mathbf{e}^{\mathbf{y}}+\frac{\mathbf{1}}{\mathbf{2}}\mathbf{a}^{\mathbf{2}}\mathbf{e}^{\mathbf{2y}}+\mathbf{b}$
8. Eliminate the arbitrary function $\mathbf{f}$ from the equation $\mathbf{z}=\mathbf{x}y+\mathbf{f}\left(\mathbf{x}^{\mathbf{2}}+\mathbf{y}^{\mathbf{2}}\right)$.
9. Define reducible and irreducible equation.
10. Define quassi linear Partial Differential Equation of second order
11. Define regular and irregular singular point.
12. Define solution matrix.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $\mathbf{P_n}\left(\mathbf{t}\right)$ and $\mathbf{P_m}(\mathbf{t})$ are Legendre Polynomials then

    Prove that $\int_{-1}^{1} \mathbf{P_n}\left(\mathbf{t}\right)\mathbf{P_m}\left(\mathbf{t}\right)\mathbf{d}t=\mathbf{0}$ if $\mathbf{m} \neq \mathbf{n}$.
14. Show that a solution matrix $\mathbf{\Phi}$ of $\mathbf{X}^{'}=\mathbf{A}\left(\mathbf{t}\right)\mathbf{X}, \mathbf{t} \in \mathbf{I}$ on I is a fundamental matrix of $\mathbf{x}^{'}=\mathbf{A}\left(\mathbf{t}\right)\mathbf{x}$ $\mathbf{o}n$ $\mathbf{I}$ if and only if $\mathbf{d}et\phi\left(\mathbf{t}\right) \neq \mathbf{0}$, $\mathbf{f}or$ $\mathbf{t} \in \mathbf{I}$.
15. Find the first three approximation for the equation $\mathbf{x}^{'}=-\mathbf{x}, \ \mathbf{x}\left(\mathbf{0}\right)=\mathbf{1}$.
16. Find the surface which is orthogonal to the one parameter system $\mathbf{z}=\mathbf{c}xy(\mathbf{x}^{\mathbf{2}}+\mathbf{y}^{\mathbf{2}})$ and which passes through the hyperbola $\mathbf{x}^{\mathbf{2}}-\mathbf{y}^{\mathbf{2}}=\mathbf{a}^{\mathbf{2}}, \mathbf{z}=\mathbf{0}$

17. If $\mathbf{u_1, u_2, \ldots u_n}$ are solutions of the homogeneous linear partial differential equation $\mathbf{F}\left(\mathbf{D}, \mathbf{D}'\right)\mathbf{z=0}$ then prove that $\sum_{\mathbf{r=1}}^{\mathbf{n}} \mathbf{C_r u_r}$ $\mathbf{w}here$ $\mathbf{C_r}'\mathbf{s}$ are arbitrary constants is also a solution.
18. Show that $\frac{\mathbf{d}}{\mathbf{d}t}\left[\mathbf{t^p J_p(t)}\right] = \mathbf{t^p J_{p-1}(t)}$.
19. Find $\mathbf{e}^{\mathbf{A}t}$ when $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{3} \end{bmatrix}$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. (a) Prove that the Legendre polynomial $\mathbf{P_n(t)} = \frac{\mathbf{1}}{\mathbf{2^n n!}}\frac{\mathbf{d^n}}{\mathbf{d}t^{\mathbf{n}}}\left(\mathbf{t^2 - 1}\right)^{\mathbf{n}}$
    (b) Find the first three Legendre polynomials.
21. State and prove the existence and uniqueness theorem for system of linear Differential Equation.
22. State and prove Picard's theorem.
23. Determine the characteristics of the equation $\mathbf{z = p^2 - q^2}$ and find the integral surface which passes through the parabola $\mathbf{4z + x^2 = 0, y = 0}$.
24. Reduce the Partial Differential Equation $\mathbf{y^2}\frac{\partial^2 \mathbf{z}}{\partial \mathbf{x^2}} - \mathbf{2x}y\frac{\partial^2 \mathbf{z}}{\partial \mathbf{x}\partial \mathbf{y}} + \mathbf{x^2}\frac{\partial^2 \mathbf{z}}{\partial \mathbf{x^2}} = \frac{\mathbf{y^2}}{\mathbf{x}}\frac{\partial \mathbf{z}}{\partial \mathbf{x}} + \frac{\mathbf{x^2}}{\mathbf{y}}\frac{\partial \mathbf{z}}{\partial \mathbf{y}}$ to canonical form and hence solve it.

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