PAM/CE/1001

M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
I Year I Semester
Core Elective
PROBABILITY AND DISTRIBUTIONS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. State the principle of Inclusion-Exclusion.
2. If A and B are independent then prove that A and B^c are independent.
3. Define indicator function.
4. Let X be an random variable with probability density function $\mathbf{f}(\mathbf{x}) = \begin{cases} \frac{2}{x^3}, & \mathbf{x} \geq 1 \\ \mathbf{0}, & \mathbf{x}<1 \end{cases}$ find E(x) and E(X^2) if exists.
5. Define median of a random variable X.
6. Define joint probability mass function of an random variable (X,Y).
7. Define continuous two dimensional random variable (X, Y).
8. Define marginal probability distribution function of random variables X and Y.
9. Define negative binomial distribution.
10. Define hypergeometric random variable.
11. Define convergence in probability.
12. State Lindeberg-Levy central limit theorem.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. **State and prove Bonferroni's inequality.**
14. Let X have the triangular probability distribution function $\mathbf{f}(\mathbf{x}) = \begin{cases} \mathbf{x}, & \mathbf{0} \leq \mathbf{x} \leq \mathbf{1} \\ \mathbf{2}-\mathbf{x}, & \mathbf{1} \leq \mathbf{x} \leq \mathbf{2} \end{cases}$ find $\mathbf{P}(\mathbf{0.3}<\mathbf{X} \leq \mathbf{1.5})$.
15. Let X be an random variable on a probability space $(\boldsymbol{\Omega}, \mathcal{S}, \mathbf{P})$. Let $\mathbf{E}|\mathbf{X}|^{\mathbf{k}}<\infty$. Prove that $\mathbf{n}^{\mathbf{k}}\mathbf{P}\{|\mathbf{X}|>\mathbf{n}\} \to \mathbf{0}$ as $\mathbf{n} \to \infty$.
16. Let $\beta_{\mathbf{n}}=\mathbf{E}|\mathbf{X}|^{\mathbf{n}}<\infty$. Prove that for arbitrary *k*, $\mathbf{2} \leq \mathbf{k} \leq \mathbf{n}$, $\beta_{\mathbf{k}-\mathbf{1}}^{\mathbf{1}/(\mathbf{k}-\mathbf{1})} \leq \beta_{\mathbf{k}}^{\mathbf{1}/\mathbf{k}}$.

17. Obtain the mean and variance of a standard Cauchy distribution truncated at both ends, with relevant space of variation as (-*c*, *c*).

18. Let X and Y be independent random variable's with probability mass functions $\mathbf{P}(\lambda_1)$ and $\mathbf{P}(\lambda_2)$ respectively. Prove that the conditional distribution of X given X + Y is binomial.

19. Let X be a random variable and $\{\mathbf{X_n}\}$ be a sequence of random variables defined on some probability space $(\mathbf{\Omega}, \mathcal{S}, \mathbf{P})$ then prove that $\mathbf{X_n} \overset{\mathbf{P}}{\to} \mathbf{X} \Rightarrow \mathbf{X_n} \overset{\mathbf{L}}{\to} \mathbf{X}$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let $\{\mathbf{A_n}\}$ be a non-decreasing sequence of events in S, that is $\mathbf{A_n} \in \mathbf{S}$, $\mathbf{n{=}1, 2, \ldots}$ and $\mathbf{A_n} \supseteq \mathbf{A_{n+1}}$, $\mathbf{n{=}2, 3, \ldots}$. Prove that $\lim\limits_{\mathbf{n}\to\infty} \mathbf{PA_n} = \mathbf{P}\left(\lim\limits_{\mathbf{n}\to\infty} \mathbf{A_n}\right) = \mathbf{P}\left(\bigcup\limits_{\mathbf{n=1}}^{\infty} \mathbf{A_n}\right)$, also prove that if $\{\mathbf{A_n}\}$ be a non-increasing sequence of events in S,
$$\lim_{\mathbf{n}\to\infty} \mathbf{PA_n} = \mathbf{P}\left(\lim_{\mathbf{n}\to\infty} \mathbf{A_n}\right) = \mathbf{P}\left(\bigcap_{\mathbf{n=1}}^{\infty} \mathbf{A_n}\right).$$

21. Let X be a non-negative random variable with distribution function F. Prove that $\mathbf{E}(\mathbf{X}) = \int_{\mathbf{0}}^{\infty} [\mathbf{1} - \mathbf{F}(\mathbf{x})]\, \mathbf{d}x$.

22. Let X and Y be independent and identically distributed random variables. Prove that $\mathbf{X} \;--\; \mathbf{Y}$ is a symmetric random variable.

23. Let X be an random variable defined on [0,1], if $\mathbf{P}(\mathbf{x} \leq \mathbf{X} \leq \mathbf{y})$ depends only on the length $\mathbf{y} - \mathbf{x}$ for all $\mathbf{0} \leq \mathbf{x} \leq \mathbf{y} \leq \mathbf{1}$. Prove that X is $\mathbf{U}(\mathbf{0}, \mathbf{1})$, $(\mathbf{P}\{\mathbf{X{=}0}\} = \mathbf{0})$.

24. Let $\{\mathbf{X_n}\}$ be any sequence of random variable's with $\mathbf{Y_n} = \frac{\mathbf{1}}{\mathbf{n}} \sum\limits_{\mathbf{k=1}}^{\mathbf{n}} \mathbf{X_k}$. Prove that a necessary and sufficient condition for the sequence $\{\mathbf{X_n}\}$ to satisfy the weak law of large numbers is that $\mathbf{E}\left\{\frac{\mathbf{y_n^2}}{\mathbf{1} + \mathbf{y_n^2}}\right\} \to \mathbf{0}$ **a**s $\mathbf{n} \to \infty$.

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15. Let X be an random variable on a probability space $(\mathbf{\Omega}, \mathcal{S}, \mathbf{P})$. Let $\mathbf{E}|\mathbf{X}|^{\mathbf{k}}<\infty$. Prove that $\mathbf{n^k P}\left\{|\mathbf{X}|>\mathbf{n}\right\} \to \mathbf{0}$ as $\mathbf{n} \to \infty$.
16. Let $\beta_{\mathbf{n}}=\mathbf{E}|\mathbf{X}|^{\mathbf{n}}<\infty$. Prove that for arbitrary *k*, $\mathbf{2} \leq \mathbf{k} \leq \mathbf{n}$, $\beta_{\mathbf{k-1}}^{\mathbf{1/(k-1)}} \leq \beta_{\mathbf{k}}^{\mathbf{1/k}}$.

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