B.Sc. DEGREE EXAMINATION,NOVEMBER 2018
I Year II Semester
Core Major - Paper IV
INTEGRAL CALCULUS AND FOURIER SERIES

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Evaluate $\int \mathbf{x^2 e^{-x} d}x$
2. Evaluate $\int \mathbf{s}in^{\mathbf{6}}\mathbf{x}\ \mathbf{c}os^{\mathbf{3}}\mathbf{x}\ \mathbf{d}x$
3. Evaluate $\int_{\mathbf{0}}^{\mathbf{a}} \int_{\mathbf{0}}^{\mathbf{b}} \left(\mathbf{x^2}+\mathbf{y^2}\right) \mathbf{d}xdy$
4. Define triple integral
5. Prove that $\mathbf{\Gamma}\left(\frac{\mathbf{1}}{\mathbf{2}}\right) = \sqrt{\pi}$
6. Evaluate $\int_{\mathbf{0}}^{\mathbf{1}} \mathbf{x^7}(\mathbf{1}-\mathbf{x})^{\mathbf{8}}\mathbf{d}x$
7. Find the constant $\mathbf{a_0}$ and $\mathbf{a_n}$ of the Fourier series for the function $\mathbf{f}(\mathbf{x}) = \mathbf{x}$ in $-\pi < \mathbf{x} < \pi$
8. Find $\mathbf{b_n}$ in the expansion of $\mathbf{x^2}$ as a fourier series in $(-\pi, \pi)$
9. Find a sine series for the function $\mathbf{f}(\mathbf{x}) = \mathbf{c}$ in $(\mathbf{o}, \pi)$
10. Write the half range of cosine series formula.
11. Prove that $\beta(\mathbf{m}, \mathbf{n})\, \beta(\mathbf{m}+\mathbf{n}, \mathbf{p}) = \frac{\mathbf{\Gamma}(m)\mathbf{\Gamma}(n)\mathbf{\Gamma}(p)}{\mathbf{\Gamma}(\mathbf{m}+\mathbf{n}+\mathbf{p})}$
12. Evaluate $\int_{\mathbf{0}}^{\mathbf{1}} \mathbf{x}(\mathbf{1}-\mathbf{x})^{\mathbf{10}}\mathbf{d}x$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Evaluate $\int \mathbf{c}os^{\mathbf{n}}\mathbf{x}\ \mathbf{d}x$ (n being a positive integer)
14. Evaluate $\int \int (\mathbf{x^2}+\mathbf{y^2})\mathbf{d}xdy$ over the region for which $\mathbf{x}, \mathbf{y}$ are each $= \mathbf{0}$ and $\mathbf{x}+\mathbf{y} = \mathbf{1}$

15. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7\theta\cos^5\theta \ \ \mathrm{d}\theta$
16. Express $\mathbf{f}(\mathbf{x}) = \mathbf{x}(-\pi < \mathbf{x} < \pi)$ as a fourier series with period 2π
17. Express $\mathbf{f}(\mathbf{x}) = \mathbf{c} - \mathbf{x}$ where $\mathbf{0} < \mathbf{x} < \mathbf{c}$ as a half range cosine series with period 2c.
18. Evaluate $\int \mathbf{x^3}\cos\mathbf{2}x \ \ \mathrm{d}x$
19. Evaluate $\iint \mathbf{x}y \ \mathrm{d}x \ \mathrm{d}y$ taken over the positive quadrant of the circle $\mathbf{x^2} + \mathbf{y^2} = \mathbf{a^2}$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Find a reduction formula for $\int \mathbf{x^m}(\log\mathbf{x})^{\mathbf{n}}\mathrm{d}x$ (where $\mathbf{m}$ and $\mathbf{n}$ are positive integers). And also evaluate $\int \mathbf{x^4}(\log \mathbf{x})^{\mathbf{3}}\mathrm{d}x$
21. Evaluate $\iiint \frac{\mathrm{d}x\,dy\,dz}{(\mathbf{x}+\mathbf{y}+\mathbf{z}+\mathbf{1})^{\mathbf{3}}}$ taken over the volume bounded by the planes $\mathbf{x}=\mathbf{0}, \mathbf{y}=\mathbf{0}$, $\mathbf{z}=\mathbf{0}, \ \mathbf{x}+\mathbf{y}+\mathbf{z}=\mathbf{1}$.
22. Express $\int_0^1 \mathbf{x^m}(\mathbf{1}-\mathbf{x^n})^{\mathbf{p}}\mathrm{d}x$ in terms of Gamma functions and evaluate the integral $\int_0^1 \mathbf{x^5}\left(\mathbf{1}-\mathbf{x^3}\right)^{10}\mathrm{d}x$
23. Show that $\mathbf{x^2} = \frac{\pi^2}{3} + 4\sum_{i=0}^{8}(-1)^n\frac{\cos nx}{n^2}$ in the interval $(-\pi < \mathbf{x} < \pi)$
24. Find a fourier series with period 3 to represent $\mathbf{f}(\mathbf{x}) = \mathbf{2}x - x^3$ in the range (0, 3).

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