

B.Sc. DEGREE EXAMINATION, NOVEMBER 2018
II Year III Semester
Allied Paper -III
ALLIED MATHEMATICS -I

Time : 3 Hours

Max.marks :75

Section A ($10 \times 2 = 20$) Marks

Answer any **TEN** questions

1. Show that $(1-x)^{-n} = \frac{1}{2^n} \left[1 - \frac{(-n)}{1!}(x) + \frac{(-n)(-n-1)}{2!}(x^2) \dots \dots \right]$
 2. Show that $\log_3 e - \log_9 e + \log_{27} e - \dots \dots \dots = \frac{\log_e 2}{\log_e 3}$
 3. State Cayley-Hamilton theorem.
 4. Show that the matrix $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ is orthogonal.
 5. Express $\tan 5\theta$ in terms of $\tan \theta$.
 6. If $\frac{\sin \theta}{\theta} = \frac{2165}{2166}$ show that $\theta = 3^\circ 1'$ nearly.
 7. Using Newton's forward interpolation formula find the population for the year 1946. Given
- | | | | | | |
|----------------------------|------|------|------|------|------|
| X(yr) | 1941 | 1951 | 1961 | 1971 | 1981 |
| Y(Population in thousands) | 46 | 66 | 81 | 93 | 101 |
8. State Newton's forward and Backward interpolation formula.
 9. Prove that $\cosh^2 x - \sinh^2 x = 1$
 10. Prove that $\sinh^{-1} x = \log \left(x + \sqrt{x^2 + 1} \right)$
 11. Find the real and imaginary parts of $\cos(A + iB)$
 12. Find the sum and product of the Eigen values of the matrix $\begin{pmatrix} 1 & 2 & 1 \\ -2 & 0 & 0 \\ 4 & 5 & 0 \end{pmatrix}$

Section B ($5 \times 5 = 25$) Marks

Answer any **FIVE** questions

13. Find the sum of the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots \dots \dots$
14. Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$

15. Show that $2^5 \cos^6 \theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$.

16. Find the value of y when $x = 2$ from the following data

X	0	3	5	6	8
Y	276	460	414	343	110

17. If $\sin(A+iB) = x+iy$ then prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ and $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$

18. Find the real and imaginary parts of $\tan^{-1}(x+iy)$

19. (i) Express $\begin{pmatrix} 2 & 4 & 8 \\ 6 & 2 & 8 \\ 2 & 2 & 2 \end{pmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

(ii) Show that $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}$ is unitary.

Section C ($3 \times 10 = 30$) Marks

Answer any **THREE** questions

20. Find the sum to infinity of the series $\frac{7}{72} + \frac{7.28}{72.96} + \frac{7.28.49}{72.96.120} + \dots$

21. Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$

22. Express $\frac{\sin 7\theta}{\sin \theta}$ as polynomial in (i) $\cos \theta$ (ii) $\sin \theta$

23. Find the value of y when $x = 1.85$ and $x = 2.05$ from the following data

X	1.7	1.8	1.9	2.0	2.1
Y	5.474	6.050	6.686	7.389	8.166

24. If $\tan(\theta+i\phi) = x+iy$ then prove that (i) $\frac{x}{y} = \frac{\sin 2\theta}{\sinh 2\phi}$ (ii) $\frac{1-(x^2+y^2)}{1+(x^2+y^2)} = \frac{\cos 2\theta}{\cosh 2\phi}$

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