B.Sc. DEGREE EXAMINATION,NOVEMBER 2018
I Year I Semester
Allied Paper -I
ALLIED MATHEMATICS -I

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Prove that $\frac{e^2+1}{e^2-1} = \frac{1+\frac{1}{2!}+\frac{1}{4!}+\cdots}{1+\frac{1}{3!}+\frac{1}{5!}+\cdots}$.
2. Show that $\left(\frac{2n}{n^2+1}\right)+\frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3+\frac{1}{5}\left(\frac{2n}{n^2+1}\right)^5+\cdots\cdots = \log_e\left(\frac{n+1}{n-1}\right)$.
3. If $y = \cos(ax+b)$, find y_n.
4. State Leibnitz formula for finding $D^n(uv)$.
5. Find $\frac{du}{dt}$, where $u = x^2+y^2+z^2$, $x = e^t, y = e^t \sin t, z = e^t \cos t$
6. Find the minimum value of $x^2+y^2+6x+12$.
7. Write the expansion of $\sin n\theta$ in powers of $\sin\theta$ and $\cos\theta$.
8. Prove that $8\cos^4\theta = \cos 4\theta + 4\cos 2\theta + 6$.
9. Find the reduction formula for $\int x^n e^{ax} dx$.
10. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.
11. Write the expansion of (i) $(1-x)^{\frac{-p}{q}}$ (ii) $\log(1+x)$, $|x| < 1$.
12. If $x = r\cos\theta$,$y = r\sin\theta$, find$\frac{\partial(x,y)}{\partial(r,\theta)}$.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the sum of the series $1+\frac{3}{4}+\frac{3\cdot 5}{4\cdot 8}+\frac{3\cdot 5\cdot 7}{4.\cdot 8\cdot 12}+\cdots$
14. Find the n^{th} derivative of $\frac{1}{(2x-1)(2x-3)(2x-5)}$.
15. If $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$ find $\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)}$.

16. Prove that $\frac{\sin 6\theta}{\sin \theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$.
17. Find the reduction formula for $\int x^m (\log x)^n dx$.
18. Sum to infinity the series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \cdots$
19. Find the maximum and minimum values of the function $f = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. a.) Show that $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \cdots\cdots\infty = \log 2$.

    b.)Show that $\log_3 e - \log_9 e + log_{27} e \cdots\cdots\infty = \frac{\log 2}{\log 3}$.
21. Find n^{th} derivative of $\sin^4 x \cos^3 x$.
22. Find the maximum and minimum values of the function $f(x, y) = x^2y^2(6 - x - y)$.
23. Prove that $-64\sin^7\theta = \sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta$.
24. If $I_{m,n} = \int_0^{\frac{\pi}{2}} \cos^m x \cos^n x dx$, prove that $I_{m,n} = \frac{m}{m+1} I_{m-1,n-1}$, Hence prove that $I_{m,m} = \frac{\pi}{2^{m+1}}$.

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