

B.Sc. DEGREE EXAMINATION, NOVEMBER 2018
I Year II Semester
Allied Paper -II
ALLIED MATHEMATICS -II

Time : 3 Hours

Max.marks :75

Section A ($10 \times 2 = 20$) Marks

Answer any **TEN** questions

1. Define characteristic function.
2. Justify : Set of all integers (\mathbb{Z}) is countable.
3. Justify : A divergent sequence may have convergent subsequence.
4. Justify : Every bounded sequence is convergent.
5. If $f : A \rightarrow \mathbb{R}$, has a derivative at a point $c \in \mathbb{R}$, then prove that f is continuous at 'c'
6. State law of the mean.
7. Find $L(\sin^2 t)$
8. State first shifting theorem of Laplace transformation.
9. Find $L^{-1} \left(\frac{s-3}{(s-3)^2 + 4} \right)$
10. Find $L^{-1} \left(\frac{s}{(s+2)^2} \right)$
11. Give an example of a countable bounded subset A of \mathbb{R} whose GLB and LUB are both in $\mathbb{R}-A$.
12. Define limit of a sequence.

Section B ($5 \times 5 = 25$) Marks

Answer any **FIVE** questions

13. If $f : A \rightarrow B$ and if $X, Y \subset B$ then prove that $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$
14. If $\sum_{n=1}^{\infty} a_n$ is convergent series, then prove that $\lim_{n \rightarrow \infty} a_n = 0$, Hence, deduce that $\sum_{n=1}^{\infty} \frac{(1-n)}{1+2n}$ is divergent.

15. State and prove Rolle's theorem.

16. Find. $L \left[\int_0^t \frac{e^{-t} \sin t}{t} dt \right]$

17. Find $L^{-1} \left(\frac{1}{s(s+1)(s+2)} \right)$

18. Prove that the set $[0, 1] = \{x/0 \leq x \leq 1\}$ is uncountable.

19. Prove that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} \right)$ is divergent.

Section C ($3 \times 10 = 30$) Marks

Answer any **THREE** questions

20. Prove that countable union of countable set is countable. Hence, deduce that set of all rational number is countable.

21. Prove that a monotonically increasing sequence which is bounded above is convergent.

22. State and prove Taylor's formula with integral form of the remainder.

23. Find

a). $L(t.e^{-t} \sin t)$

b). $L \left(\frac{e^{-3t} - e^{-4t}}{t} \right)$

24. Find

a). $L^{-1} \left[\log \left(\frac{1+s}{s^2} \right) \right]$

b). $L^{-1} [\tan^{-1}(s+1)]$

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