B.Sc. DEGREE EXAMINATION,NOVEMBER 2018
III Year V Semester
Core Major - Paper X
REAL ANALYSIS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Prove that $A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ...\right\}$is a countable set.
2. Justify : Every bounded sequence is a convergent sequence.
3. Find $\lim\inf_{n\to\infty}(-1)^n$ and $\lim\sup_{n\to\infty}(-1)^n$
4. When a series $\sum_{n=1}^{\infty} a_n$ is said to be converges conditionally?
5. If ρ is a metric on M, is ρ^2 metric on M ?
6. If G_1 and G_2 are open subsets of a metric space M, then prove that $G_1 \cap G_2$ is open
7. Define connected set.
8. Define contraction mapping.
9. State chain rule.
10. If $f : A \to R$ has a derivative at a point $c \in R$ then prove that f is continuous at c.
11. Justify : A divergent sequence may have a convergent sub sequence.
12. Find LUB and GLB of $\{\pi + 1, \pi + {}^1\!/_2, \pi + {}^1\!/_3 ....\}$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove : Countable union of countable sets is countable.
14. Prove that the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is converges if and only if it is Cauchy.
15. Prove that the continuous function of a continuous function is continuous.
16. If the subset A of the metric space (M, ρ) is totally bounded, then prove that A is bounded. .
17. State and prove Rolle's theorem.

18. Prove that f is continuous if and only if the inverse image of every open set is open.
19. State LUB axiom. Using it, prove that if A is any non empty subset of R that is bounded below, then A has GLB in R

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that a increasing sequence which is bounded above is convergent. Hence, deduce $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.
21. If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that

    i) $a_1 \geq a_2 \geq ... \geq a_n \geq a_{n+1} \geq ...$ and

    ii) $\lim_{n\to\infty} a_n = 0$, then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ convergent.

22. Let (M, γ) be a metric space and **a** be a point in M. Let $f$and g are real valued functions whose domains are subsets of M. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = N$, then prove that

    (a.) $\lim_{x\to a} [f(x)g(x)] = LN$

    (b.) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{N}, ifN \neq 0.$

23. State and prove nested interval theorem.
24. State and prove second fundamental theorem of calculus.

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