B.Sc. DEGREE EXAMINATION,NOVEMBER 2018
III Year V Semester
Core Major - Paper XII
GRAPH THEORY

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. When two graphs are said to be isometric?
2. (i) Define cut vertex in a graph (ii) Define walk of a graph.
3. Define Hamiltonian path and Hamiltonian cycle of graph.
4. Define Weighted graph.
5. Define complete bipartite graph.
6. Define adjacency matrix of graph.
7. Define planar graph.
8. Define dual of plane graph.
9. Define k-edge colouring of a graph.
10. Define the chromatic number of a graph.
11. Define subgraph.
12. Define Eulerian trail of the graph.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that every $(p, q)-graph$ with $q \geq p$ contains a cycle.

14. If G is (p, q) -graph with $p \geq 3\ and\ q \geq \dfrac{p^2-3p+6}{2}$ then prove that G is Hamiltonian.

15. For a $(p, q)-graph$ G, prove that the following statements are equivalent (a) G is a tree (b) G is connected and $q = p-1$ (c) G is acyclic and $q = p-1$

16. Prove that a graph is planar if it contains no contraction $K_5\ or\ K_{33}$.

17. If G is a $(p, q)-graph$ then prove that $\chi(G) \geq \dfrac{p^2}{p^2-2q}$

18. Prove that a graph G on p vertices is connected if and only if $(A+I)^{p-1}$ has no zero entries.

19. Prove that every nontrivial graph contains at least two vertices which are not cut-vertices.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $q > \dfrac{p^2}{4}$ then prove that every $(p, q)-graph$ contains a triangle.

21. Prove that a nontrivial connected graph is Eulerian if and only if it has no vertex of odd degree.

22. Prove that a $(p, q)-graph$ G is a bipartite graph if and only if it contains no odd cycles.

23. State and prove Euler formula for planar graphs.

24. If G is a graph on p vertices then prove that

(a) $2\sqrt{p} \leq \chi(G) + \chi(\overline{G}) \leq p+1$ (b) $p \leq \chi(G)\chi(\overline{G}) \leq \dfrac{(p+1)^2}{4}$

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