B.Sc. DEGREE EXAMINATION,NOVEMBER 2018
III Year VI Semester
Core Major - Paper XIII
LINEAR ALGEBRA

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define vector space.
2. Define Homomorphism.
3. Define dimension.
4. Define dual space.
5. Define inner product space
6. Define orthogonal complement.
7. Define linear transformation.
8. Define regular.
9. Define similar.
10. Define invariant.
11. Define Matrix of T
12. Compute the matrix product $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}^2$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that V is isomorphic to the external direct sum of $U_1, ...U_n$ if V is the internal direct sum of $U_1, ...U_n$.
14. Show that L(S) is a subspace of V.
15. Prove that if dim_F V = m then dim_F Hom(V,F) =m.
16. Prove that T ϵ A(V) is singular if and only if there exists a v #0 in V such that vT = 0 when V is finite dimension over F.
17. Prove that T is regular if and only if T maps V onto V where V is the finite dimensional over F.
18. Show that T satisfies a polynomial of degree n over F where V is n - dimensional over F and if T ϵ A(V) has all its characteristic roots in F.

19. Show that If V is n-dimensional over F and if T ϵ A(V) has the matrix m_1 (T) in the basis $v_1, v_2, ..., v_n$ and the matrix m_2 (T) in the basis $w_1, w_2, ..., w_n$ of V over F then there is an element C ϵ Fn such that $m_2(T) = Cm_1(T)C^{-1}$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that if v is a finite dimensional vector space and if $u_1, ..., u_n$ Span V then some subset of $u_1, ..., u_n$ forms a basis of V
21. Show that W is finite dimensional, dim W $\leq$ dim V and
dim V/W = dim V - dim W Where V is finite- dimensional and W is a subspace of V.
22. Prove that Hom (V, W) is of dimension mn over F if V and W are dimensions m and n respectively over F.
23. Prove that V has an Orthonormal set as basis where V is the finite- dimensional inner product space.
24. If λ ϵ F is a characteristic root of T ϵ A(V) then λ is a root of the minimal polynomial of T. In particular Prove that T, only has a finite number of characteristic roots in F.

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