B.Sc. DEGREE EXAMINATION,APRIL 2019
III Year VI Semester
Stochastic Processes

Time : 3 Hours **Max.marks :60**

Section A $(10 \times 1 = 10)$ Marks

Answer any **TEN** questions

1. Define Random Process.
2. Define Process with Independent Increment.
3. Show the Markov chain as a digraph.
4. Define Order of a Markov Chain.
5. State the correlation coefficient of the Poisson process.
6. Show the second order Poisson process.
7. What is linear growth process?
8. Define pure birth process.
9. Show the balance equations of steady state birth and death process.
10. Write the Little's formulae for Ls and L$_q$.
11. What is a stationary process?
12. Explain the reflecting barriers.

Section B $(5 \times 4 = 20)$ Marks

Answer any **FIVE** questions

13. Classify the Stochastic Process with suitable example.
14. Prove that Markov chain is completely determined if the initial state probability distribution and one step Transition Probabilities are known.
15. Prove that , for a Poisson process $N(t)$;

    $$P[t < \gamma < t + dt/N(T) = 1] = \frac{dt}{T};\ 0 < t < T$$
16. Derive the differential equations of pure birth process using state flow diagram.
17. For M/M/1:∞/FIFO queuing model, find the expected number of customers in the system.
18. Prove that sum of two independent Poisson process is also a Poisson process.
19. List the postulates of birth and death process.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. "Derive the Poisson Process stating the necessary assumption clearly".

21. The one step T.P.M of a Markov chain $(X_n\,;\ n = 0,\,1,\,2,\,....)$ having state space $S = (1,\,2,\,3)$ is $\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $\pi_0 = (0.7,\,0.2,\,0.1)$

    Find (i) $P\,(X_2 = 3/\,X_0 = 1)$ (ii) $P\,(X_3 = 2,\,X_2 = 3,\,X_1 = 3,\,X_0 = 2)$

22. Suppose that customers arrive at a counter according to a Poisson process with mean rate of 2 per minute. Find the probability that the interval between two successive arrivals is

    (i.) More than 1 minute

    (ii.) 4 minutes or less

    (iii.) Between 1 and 2 minutes.

23. Define Yule - Fury process and derive its probability distribution as Geometric distribution.

24. At a railway station, only one train is handled at a time. The railway yard is sufficient only for 2 trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them at an average rate of 6 per hour. Assuming Poisson arrivals and exponential service distribution, find the probabilities for the number of trains in the system. Also find the average waiting time of a new train coming into the yard. If the handling rate is doubled, how will the above results get modified?

B.Sc. DEGREE EXAMINATION,APRIL 2019
III Year VI Semester
Stochastic Processes

Time : 3 Hours **Max.marks :60**

Section A $(10 \times 1 = 10)$ Marks

Answer any **TEN** questions

1. Define Random Process.
2. Define Process with Independent Increment.
3. Show the Markov chain as a digraph.
4. Define Order of a Markov Chain.
5. State the correlation coefficient of the Poisson process.
6. Show the second order Poisson process.
7. What is linear growth process?
8. Define pure birth process.
9. Show the balance equations of steady state birth and death process.
10. Write the Little's formulae for Ls and L_q.
11. What is a stationary process?
12. Explain the reflecting barriers.

Section B $(5 \times 4 = 20)$ Marks

Answer any **FIVE** questions

13. Classify the Stochastic Process with suitable example.
14. Prove that Markov chain is completely determined if the initial state probability distribution and one step Transition Probabilities are known.
15. Prove that , for a Poisson process $N(t)$;

    $$P[t < \gamma < t + dt/N(T) = 1] = \frac{dt}{T};\ 0 < t < T$$
16. Derive the differential equations of pure birth process using state flow diagram.
17. For M/M/1:∞/FIFO queuing model, find the expected number of customers in the system.
18. Prove that sum of two independent Poisson process is also a Poisson process.
19. List the postulates of birth and death process.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. "Derive the Poisson Process stating the necessary assumption clearly".

21. The one step T.P.M of a Markov chain $(X_n;\ n = 0, 1, 2, ...)$ having state space $S = (1, 2, 3)$ is $\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $\pi_0 = (0.7, 0.2, 0.1)$

    Find (i) $P(X_2 = 3/ X_0 = 1)$ (ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

22. Suppose that customers arrive at a counter according to a Poisson process with mean rate of 2 per minute. Find the probability that the interval between two successive arrivals is

    (i.) More than 1 minute

    (ii.) 4 minutes or less

    (iii.) Between 1 and 2 minutes.

23. Define Yule - Fury process and derive its probability distribution as Geometric distribution.

24. At a railway station, only one train is handled at a time. The railway yard is sufficient only for 2 trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them at an average rate of 6 per hour. Assuming Poisson arrivals and exponential service distribution, find the probabilities for the number of trains in the system. Also find the average waiting time of a new train coming into the yard. If the handling rate is doubled, how will the above results get modified?