M.Sc DEGREE EXAMINATION,APRIL 2019
I Year I Semester
Algebra - I

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Normalizer of an element in a group.
2. State Cauchy's theorem.
3. Define an R-module.
4. Define internal direct product on group.
5. Define the trace of a matrix.
6. Define a skew-symmetric matrix.
7. State Wedderburn theorem.
8. Define primitive root of p.
9. Define a solvable group.
10. Define the adjoint of x.
11. Define cyclic R-module.
12. Define normal transformation.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $o(G) = p^2$ where p is a prime number then prove that G is abelian.
14. If G is a solvable group and if $\bar{G}$is a homomorphic image of G then prove that $\bar{G}$ is solvable.
15. If (υT, υT) = (υ,υ) for all υ∈V, prove that T is unitary.
16. Prove that any two finite fields having the same number of elements are isomorphic
17. Prove that *N(xy) = N(x) N(y)*, for all $x, y \in Q$
18. Prove that conjugacy is an equivalence relation.
19. If T ∈A(V) is Hermitian then prove that all its characteristics roots are real.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove first part of Sylow's theorem.
21. Show that S_n is not solvable for $n = 5$.
22. If T $\in$A(V) then prove that

    a) T * $\in$A(V)

    b) (T *)* = T

    c) (ST)* = T *S*

    d) (S+T)* = S* + T *.
23. Let F be a finite field with q elements and suppose that F$\subset$K where K is also a finite field then prove that K has q^n elements where n = [K : F].
24. Let C be the field of complex numbers and suppose that the division ring D is algebraic over C then prove that D = C

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