M.Sc DEGREE EXAMINATION,APRIL 2019
I Year II Semester
Algebra II

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. When is an element said to algebraic over F?
2. Define Algebraic extension.
3. Define root of multiplicity m.
4. Define simple extension.
5. Define group of automorphisms.
6. Define Normal extension.
7. Define the invariant subspace of a linear transformation.
8. If S and T are nilpotent linear transformations which commute, prove that S + T is nilpotent.
9. Define companion matrix of f(x).
10. Define rational canonical form.
11. When is a field K said to be an extension of field F?
12. Define the index of nilpotence of a linear transformation.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.
14. If p(x) $\in$ F[x] and if K is an extension of F,then for any element b $\in$ K, Prove that p(x) =(x-b) q(x) + p(b), where q(x) $\in$ K[x] and where deg q(x) = deg p(x) − 1.
15. Prove that the fixed field of G is a sub-field of K.
16. If V is a n-dimensional vector space over F and if T$\in$ A(V) has all its characteristic roots in F. Then T satisfies a polynomial of degree n over F.
17. Prove that two nilpotent linear transformations are similar if and only if they have same invariants.

18. Suppose that V $= V_1 \oplus V_2$ where V_1 and V_2 are subspaces of V invariant under T. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2 respectively. If the minimal polynomial of T_1 over F is P_1(x) while that of T_2 is P_2(x), then prove that the polynomial for T over F is the least common multiple of P_1(x) and P_2(x).
19. Let $dim\ M = m$, if M is cyclic with respect to T then prove that $dim\ M\ T^k = m - k$, for all $k \leq m$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the number e is transcendental.
21. If F is of characteristic 0 and if a,b are algebraic over F. Prove that there exists an element c $\in$ F(a,b) such that F(a,b) $= F(c)$.
22. Show that if K is a extension of F then G(K,F) is a finite group and O[G(K,F)] $\leq$ [K;F].
23. Prove that there exists a subspace W of V, invariant under T such that V $= V_1$ $\oplus$ W.
24. Let V and W be two vector spaces over F and suppose that ψ is a vector space isomorphism of V onto W. Suppose that S $\in A_F$(V) and T $\in A_F$(W) are such that for v$\in V$, (vS)$\psi =$ (vψ)T. Prove that S and T have the same elementary divisors.

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