M.Sc DEGREE EXAMINATION,APRIL 2019
II Year III Semester
Differential Equations

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Analytic functions
2. Define regular singular point.
3. Define fundamental matrix.
4. Represent the system of equations in vector form $x'_1 = 5x_1 - 2x_2; \quad x'_2 = 2x_1 + x_2$
5. State Cauchy-Peano theorem.
6. State Lipschitz condition.
7. Eliminate the arbitrary constants a and b from the equation $2z = (ax + y)^2 + b$
8. Eliminate the arbitrary function f from the relation $z = xy + f\left(x^2 + y^2\right)$
9. Solve $\left(D^2 - 5DD' + 6D'^2\right) z = 0$
10. Classify $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$
11. Find the particular Integral of $\left(D^2 - 5DD' + 6D'^2\right) z = e^{x+y}$
12. Find the power series solution of the differential equation $x' + x = t,\ x(0) = 0$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $P_m(t)$ and $P_n(t)$ are Legendre polynomials then prove that $\int_{-1}^{1} P_m(t) P_n(t) = 0$ if $m \neq n$
14. Determine the fundamental matrix for $x' = Ax\ where\ A = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$
15. By the method of successive approximations solve the initial value problem $x' = -x,\ x(0) = 1, t \geq 0$
16. Find the general solution of the partial differential equation $px(x + y) = qy(x + y) - (x - y)(2x + 2y + z)$

17. Solve $\left(D^2 + 3DD' + 2D'^2\right) z = x + y$

18. Show that the equation $\dfrac{\partial^2 y}{\partial t^2} + 2k\dfrac{\partial y}{\partial t} = c^2\dfrac{\partial^2 y}{\partial x^2}$ possesses solutions of the form $\sum_{r=0}^{\infty} C_r e^{-kt} cos\left(a_r x + \in_r\right) cos\left(w_r t + \delta_r\right)$ where $C_r, \alpha_r, \in_r, \delta_r$ are constants and $w_r^2 = \alpha_r^2 c^2 - k^2$

19. State and prove Gronwall inequality

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Let $A_1,\ A_2, \ldots..$ be the positive zeros of the Bessel function $J_p(t)$then prove that
$$\int_0^1 tJ_p\left(A_m t\right) J_p\left(A_n t\right) dt = \begin{cases} 0 \; if \; m \neq n \\ \frac{1}{2} J_{P+1}(A_n)^2 \; if \; m = n \end{cases}$$

21. State and prove Existence and Uniqueness theorem in systems of linear differential equations.

22. State and prove Picard's theorem.

23. Use Charpit's method to solve the partial differential equation $\left(p^2 + q^2\right) y = qz$

24. Reduce the partial differential equation $y^2\dfrac{\partial^2 z}{\partial x^2} - 2xy\dfrac{\partial^2 z}{\partial x \partial y} + x^2\dfrac{\partial^2 z}{\partial y^2} = \dfrac{y^2}{x}\dfrac{\partial z}{\partial x} + \dfrac{x^2}{y}\dfrac{\partial z}{\partial y}$ to canonical form and hence solve it.

M.Sc DEGREE EXAMINATION,APRIL 2019
II Year III Semester
Differential Equations

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Analytic functions
2. Define regular singular point.
3. Define fundamental matrix.
4. Represent the system of equations in vector form $x'_1 = 5x_1 - 2x_2; \quad x'_2 = 2x_1 + x_2$
5. State Cauchy-Peano theorem.
6. State Lipschitz condition.
7. Eliminate the arbitrary constants a and b from the equation $2z = (ax + y)^2 + b$
8. Eliminate the arbitrary function f from the relation $z = xy + f\left(x^2 + y^2\right)$
9. Solve $\left(D^2 - 5DD' + 6D'^2\right) z = 0$
10. Classify $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
11. Find the particular Integral of $\left(D^2 - 5DD' + 6D'^2\right) z = e^{x+y}$
12. Find the power series solution of the differential equation $x' + x = t, \ x(0) = 0$

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $P_m(t)$ and $P_n(t)$ are Legendre polynomials then prove that $\int_{-1}^{1} P_m(t) P_n(t) = 0$ if $m \neq n$
14. Determine the fundamental matrix for $x' = Ax \ where \ A = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$
15. By the method of successive approximations solve the initial value problem $x' = -x, \ x(0) = 1, t \geq 0$
16. Find the general solution of the partial differential equation $px(x + y) = qy(x + y) - (x - y)(2x + 2y + z)$

17. Solve $\left(D^2+3DD'+2D'^2\right)z=x+y$

18. Show that the equation $\dfrac{\partial^2 y}{\partial t^2}+2k\dfrac{\partial y}{\partial t}=c^2\dfrac{\partial^2 y}{\partial x^2}$ possesses solutions of the form $\sum_{r=0}^{\infty} C_r e^{-kt} cos\left(a_r x+\in_r\right) cos\left(w_r t+\delta_r\right)$ where $C_r,\alpha_r,\in_r,\delta_r$ are constants and $w_r^2=\alpha_r^2c^2-k^2$

19. State and prove Gronwall inequality

Section C $(3\times 10=30)$ Marks

Answer any **THREE** questions

20. Let $A_1,\ A_2,\ldots..$ be the positive zeros of the Bessel function $J_p(t)$then prove that
$$\int_0^1 tJ_p\left(A_m t\right)J_p\left(A_n t\right)dt=\begin{cases}0\ if\ m\neq n\\ \frac{1}{2}J_{P+1}(A_n)^2\ if\ m=n\end{cases}$$

21. State and prove Existence and Uniqueness theorem in systems of linear differential equations.

22. State and prove Picard's theorem.

23. Use Charpit's method to solve the partial differential equation $\left(p^2+q^2\right)y=qz$

24. Reduce the partial differential equation $y^2\dfrac{\partial^2 z}{\partial x^2}-2xy\dfrac{\partial^2 z}{\partial x\partial y}+x^2\dfrac{\partial^2 z}{\partial y^2}=\dfrac{y^2}{x}\dfrac{\partial z}{\partial x}+\dfrac{x^2}{y}\dfrac{\partial z}{\partial y}$ to canonical form and hence solve it.