17PAMCT4A11

M.Sc DEGREE EXAMINATION, APRIL 2019 II Year IV Semester Differential Geometry and Tensor Calculus

Time : 3 Hours

Max.marks:75

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

- 1. State Serret- Frenet formula
- 2. Define involute and evolute of a space curve.
- 3. Define a helicoid? what is the pitch of the helicoid?
- 4. Define anchor ring.
- 5. Define geodesic parallels.
- 6. Define Gaussian curvature.
- 7. Define covariant tensor of rank one.
- 8. Define symmetric and skew symmetric tensors
- 9. Define a metric tensor.
- 10. Define Christoffel 3- index symbols of the first kind
- 11. Define osculating plane.
- 12. Define geodesic for a curve.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

- 13. Calculate the curvature and torsion of the cubic curve $\bar{r} = (u, u^2, u^3)$
- 14. Show that the metric is invariant under a parameter transformation.
- 15. A particle is constrained to move on a smooth surface under no force except the normal reaction. Prove that its path is geodesic.
- 16. Prove that any linear combination of tensors of the same type and rank is again a tensor of the same type and rank

17. Prove that
$$\frac{\partial}{\partial x^i} \log \sqrt{g} = \left\{ \begin{array}{c} \alpha \\ i\alpha \end{array} \right\}$$

- 18. Prove that the characteristic property of helices is that the ratio of the curvature to the torsion is constant at all points.
- 19. Define the osculating sphere at a point on the curve and derive the formula for its centre and radius.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

- 20. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$
- 21. Find the surface of revolution which is isometric with a region of right helicoids.
- 22. State and prove Gauss Bonnet theorem
- 23. Prove that the law G of transformation of mixed tensors is transitive.
- 24. Derive Riemann- Christoffel tensors of first and second kind.

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