M.Sc DEGREE EXAMINATION,APRIL 2019
I Year I Semester
Modern Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define conjugate elements.
2. Define centre of a group.
3. Define cyclic group.
4. Define R - module.
5. Define unitary matrix.
6. Define trace and transpose of a matrix.
7. Define finite field.
8. Define division ring.
9. States Abel's theorem.
10. Define solvable by radicals.
11. Write finite sequence of fields radical.
12. Define internal direct product.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that conjugacy is an equivalence relation on G.
14. Let G be a finite abelian group prove that G is isomorphic to the direct product of its Sylow subgroup.
15. If N is a normal and AN = NA then prove that AN* =N*A.
16. Let F be a finite field then prove that F has p^m element where the prime number p is the characteristic of F.
17. Prove that if a$\in$H then a^{-1} $\in$H if and only if N(a) = 1.
18. Let C be the field of complex number and suppose that the division ring D is algebraic over C. then prove that D = C.
19. If O (G) = p^n where p is a prime number then prove that Z (G) $\neq$ e.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove first sylow's theorem.
21. Let R be a Euclidean ring then prove that any finitely generated R-module M is the direct sum of finite number of cyclic sub-module.
22. Prove that the linear transformation T on V is unitary if and only if it takes orthonormal basis of V into an orthonormal basis of v.
23. For every prime number p and for every integer m then prove that there exist a field having p^m elements.
24. If p(x) = F[x] is solvable by Radical then prove that the Galois group over F of p(x) is a solvable group.

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