M.Sc DEGREE EXAMINATION,APRIL 2019
I Year II Semester
Linear Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define extension field.
2. Define transcendental and give an example.
3. Define roots of polynomial.
4. Define simple extension of F.
5. Define distinct Automorphism.
6. Define normal extension of F.
7. Define index of nilpotence of T.
8. Define invariant.
9. Define annihilator.
10. Write the form of rational canonical form of T.
11. If S and T are nilpotent linear transformation, then S+T is nilpotent.
12. Define algebraic extension.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that the elements in K which are algebraic over F form a subfield of K.
14. If p(x) $\in$ F[x] and if k is an extension of F then for any element b$\in$k we have p(x) = (x-b) q(x) + p(b) where q(x)$\in$k[x] and deg q(x) = deg p(x) - 1.
15. Prove that the fixed field of G is a sub-field of k.
16. Prove that two nilpotent linear transformations are similar if and only if they have same invariants.
17. Let $dim\ M = m$, if M is cyclic with respect to T then prove that $dim\ M\ T^k = m - k$, for all $k \leq m$.
18. If L is a finite extension of F and K is a sub-field of L which contains F then prove that [K: F] divides [L: F].

19. For any f(x),g(x)$\in$F[X] and $\alpha \in$ F, then prove that (i) (f(x)+g(x))*= f*(x) + g*(x), (ii) (αf(x))* = αf*(x).

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If L is a finite extension of K and K is a finite extension of F, then prove that L is a finite extension of F.
21. If F is of characteristics zero and if a,b are algebraic over F then prove that there exist an element c $\in$ F(a,b) such that F(a,b) = F(c).
22. If K is a finite extension of F then prove that G (K, F) is a finite group and o(G(k,F)) $\leq$ [k : F].
23. If T $\in$ A (V) has all its characteristic roots in F then prove that there is a basis of V in which the matrix of T is triangular.
24. Prove that the elements S and T in A_F(V) are similar in A_F(V) if and only if they have the same elementary divisors.

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