

**M.Sc DEGREE EXAMINATION, APRIL 2019**  
**II Year IV Semester**  
**Differential Geometry and Tensor Calculus**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. Find the equation of the tangent surface to the curve  $r = (u, u^2, u^3)$ .
2. Define osculating plane.
3. Define direction coefficients on a surface.
4. Define double family of curves.
5. Define geodesics.
6. Write down the Canonical equations for geodesics.
7. Define contravariant tensor.
8. State Quotient Laws of tensors.
9. Define Metric tensor.
10. Define Second Covariant derivative.
11. Define Arc length between any two points of a curve.
12. Define Helicoids.

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. State and prove Serret – Frenet formula.
14. Prove that
14. Prove that  $[r', r'', r'''] = k^2 \tau$ .
15. Find a surface of a revolution which is isometric with a region of the right helicoid.
16. Find the geodesic curvature of the parametric curves  $\vartheta = c$
17. Explain Symmetric and Skew-Symmetric Tensors.
18. Prove that the sum of two tensors which have the same number of covariant and the same number of contravariant indices is again a tensor of the same type and rank as the given tensors.
19. State and Prove Ricci's theorem.

**Section C** ( $3 \times 10 = 30$ ) MarksAnswer any **THREE** questions

20. Find the Curvature and torsion of the curve of intersection of the two quadric surfaces  $ax^2 + by^2 + cz^2 = 1, a'x^2 + b'y^2 + c'z^2 = 1$ .
21. Find the coefficients of the direction which make an angle  $\frac{\pi}{2}$  with the direction whose coefficients are (l,m).
22. State and Prove Gauss-Bonnet theorem.
23. Prove that if all components of a tensor vanish in one coordinate system, then they necessarily vanish in all other admissible coordinate systems.
24. Derive the transformation law for the Christoffel's symbol of first kind.

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