M.Sc. DEGREE EXAMINATION,APRIL 2019
I Year I Semester
ALGEBRA

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. If $A = \{1, 2, 3, 4, 5\}\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$, Find $\sigma\tau$ and $\sigma\tau$.
2. If $\phi : G \to G'$is a group homorphism of G onto G' and G is abelian, Prove that $G'$is abelian.
3. Define skew field.
4. Define integral domain and give an example.
5. Define linear transformation.
6. When is a square matrix said to be diagonalizable.
7. Define *H*-unitary linear transformation.
8. Define orthonormal vectors.
9. Define extension field of a field and give an example.
10. Define primitive root of unity.
11. Prove that every subgroup of an abelian group is normal.
12. Define characteristic of a ring and give an example.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. State and prove the Theorem of Lagrange.
14. Prove that every field is an integral domain and every finite integral domain is a field.
15. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
16. Prove that every finite-dimensional Euclidean vector space has an orthonormal basis.
17. If *F* is a finite field of characteristic *p* with algebraic closure $\overline{F}$, Prove that $x^{p^n} - x$ has p^ndistinct zeros in $\overline{F}$.

18. If H is a subgroup of a group G, Prove that left coset multiplication is well defined by the equation $(aH)(bH) = (ab)H$ if and only if left and right cosets coincide so that $aH = Ha$ for all $a \in G$.

19. If E is a finite extension field of a field F and K is a finite extension field of E, Prove that K is a finite extension of F and $[K : F] = [K : E]\,[E : F]$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove Cayley's Theorem.

21. Define V/W and prove that it is a vector space.

22. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{pmatrix}$, hence find its inverse.

23. If F is a field of quotients of D and L is any field containing D, Prove that there exists a map $\psi : F \to L$ that gives an isomorphism of F with a subfield of L such that $\psi(a) = a$ for $a \in D$.

24. Prove that every field F has an algebraic closure $\overline{F}$.

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