M.Sc DEGREE EXAMINATION,APRIL 2019
II Year IV Semester
Functional Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define a Banach space and give an example.
2. Define a closed unit sphere in R^2 and show that it is convex.
3. Define a Hilbert space.
4. Define a complete orthonormal set.
5. If H is a Hilbert space,prove that if T∈B(H),then $(T^*)^* = T$.
6. Define a normal operator in a Hilbert space H.
7. Show that multiplication is jointly continuous in any Banach algebra.
8. If A is a Banach algebra,define the radical,maximal left ideal in A.
9. Define Gelfand mapping.
10. Define a Banach algebra.
11. When do we say that two normed linear spaces are isometrically isomorphic?
12. Show that parallelogram law holds in a Hilbert space.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If N is a normed linear space and x_0 is a non-zero vector in N.then prove that there exists a functional f_0 in N^* such that f_0 (x_0) $=\|x_0\|$ and $\|f_0\|$ =1.
14. If M and N are closed linear subspaces of a Hilbert space H,such that M⊥N,then show that the linear subspace M + N is also closed.
15. If T is an operator on a Hilbert space H such that T(x,x)=0 for ∀x ∈H,then prove that T =0.
16. Let A be a Banach Algebra. Let S be the set of singularities of A and Z be the set of all topological divisors of zero. Then prove that boundary of S is a subset of Z.
17. If x is a normal element in a B^* - algebra, then prove that $\left\|x^2\right\| = \|x\|^2$.
18. If 0 is the only topological divisor of zero in A,then show that A = C.

19. If N_1 and N_2 are normal operators on a Hilbert space H with the property that either commute with the adjoint of the others,then prove that $N_1 + N_2$ and N_1N_2 are normal.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove Hahn- Banach theorem.
21. State and prove open mapping theorem.
22. If H is a Hilbert space anf f is an arbitrary functional in H^*, show that there exists a unique vector y in H such that f(x) =(x , y) for every x in H.
23. With usual notations prove that r(x) = lim$\|x^n\|^{1/n}$.
24. State and prove that Gelfand-Neumark theorem.

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