M.Sc. DEGREE EXAMINATION,APRIL 2019
I Year I Semester
REAL ANALYSIS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Riemann-Stieltjes integral of a bounded real function f over [a , b].
2. Define a unit step function I.
3. What do you understand by uniform convergence of a sequence of functions $\{f_n\}$?.
4. Let $\{f_n\}$ be a sequence of functions defined on a set E.When do you say that a sequence $\{f_n\}$ is i) pointwise bounded and ii) uniformly bounded on E ?.
5. Define linear transformation of a mapping A of a vector space X into a vector space Y .
6. Define a contraction map.
7. State intermediate value theorem.
8. Define Borel sets and give an example.
9. Define Lebesgue integral of a non negative measurable function f.
10. If f is integrable over E ,A and B are disjoint measurable sets contained in E, prove that $\int_{A\cup B} f = \int_A f + \int_B f$.
11. State the theorem on integration by parts for real valued functions.
12. Define a measurable set E.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that $\displaystyle\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f d\alpha$.
14. State and prove weierstrass theorem on uniform convergence of series.
15. If $A \in L(R^n, R^m)$, then show that $\|A\| < \infty$ and A is uniformly continuous mapping of R^ninto R^m.
16. If $\mathbb{C}$ be a collection of open sets of real ,prove that there is a countable subcollection $\{O_i\}$ of $\mathbb{C}$ such that $\displaystyle\bigcup_{O\in\mathbb{C}} O = \bigcup_{i=1}^{\infty} O_i$.

17. If $\langle f_n \rangle$ is a sequence of measurable functions that converges in measure to f,show that there is a subsequence $\langle f_{n_k} \rangle$ that converges to f almost everywhere.
18. If f is continuous on [a, b] ,then prove that $f \in \Re(\alpha)$on [a, b].
19. If $E_1$and E_2 are measurable then show that $E_1 \cup E_2$ is measurable.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that $f \in \Re(\alpha)$ on [a, b] if and only if for every $\in > 0$ there exists a partition P of [a,b] such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.
21. If $\{f_n\}$is a pointwise bounded sequence of complex functions on a countable set E, then show that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$converges for every $x \in E$.
22. State and prove the inverse function theorem.
23. State and prove Heine- Borel theorem.
24. Let f be defined and bounded on a measurable set E with mE finite. Then show that in order that $\inf\limits_{f \leq \psi} \int_E \psi(x)dx = \sup\limits_{f \geq \phi} \int_E \phi(x)dx$ for all simple functions ϕ and ψ,it is necessary and sufficient that f is measurable.

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