M.Sc DEGREE EXAMINATION,APRIL 2019
I Year II Semester
Complex Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define entire function
2. What do you mean by simply connected set ?
3. Define essential singularity with an example
4. State Maximum modulus theorem
5. State Weierstrass factorisation theorem for entire functions
6. Define Gamma function
7. State Mean value theorem on harmonic functions
8. Explain Dirichiet region
9. Explain finite rank of an entire function
10. Define genus of an entire function
11. Define Convex function.
12. State Gauss's formula

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Let G be an open set which is a -star shaped. If γ_0 is the curve which is constantly equal to a then prove that every closed rectifiable curve in G is homotopic to γ_0
14. Show that $\displaystyle\int_0^{\pi} \frac{d\theta}{a + cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$ for $a > 1$.
15. If $|z| \leq 1|$ and $p \geq 0$ then prove that $|1 - E_p(Z)| \leq |Z|^{p+1}$
16. Let G and Ω be the regions such that there is a one to one analytic function f of G onto Ω: let $a \in G$ and $\alpha = f(a)$. If g_α and γ_α are the green's functions of G and Ω with singularities a and α respectively then prove that $g_\alpha(z) = \gamma_\alpha[f(z)]$
17. Let f be an analytic function on the disc B(a: r) such that
$|f'(z) - f'(a)| < |f'(a)|$ for all z in B(a: r) , $z \neq a$ a then prove that f is one to one .

18. If G is simply connected and $f : G \to G$ is analytical in G then prove that f has a primitive in G .

19. If $G = \{z : ReZ > 0\}$ and $f_n(z) = \int_{\frac{1}{n}}^{n} e^{-t}\ t^{z-1}\ dt$ for $n \geq 1$ and z in G then prove that each f_n is analytic on G and the sequence is convergent in H(G).

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove GOURSAT' s theorem

21. State and prove SCHWARZ 's lemma for analytic functions

22. Let G be a region which is not a whole plane and such that every non vanishing analytic function on G has an analytic square root .If $a \in G$ G then prove that there is an analytic function f on G such that

    (i) $f(a) = 0$ and $f'(a) > 0$

    (ii) f is one to one

    (iii) $f(G) = D = z : |Z| < 1$

23. State and prove Harnack's theroem

24. State and prove Schottky's theorem

M.Sc DEGREE EXAMINATION,APRIL 2019
I Year II Semester
Complex Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define entire function
2. What do you mean by simply connected set ?
3. Define essential singularity with an example
4. State Maximum modulus theorem
5. State Weierstrass factorisation theorem for entire functions
6. Define Gamma function
7. State Mean value theorem on harmonic functions
8. Explain Dirichiet region
9. Explain finite rank of an entire function
10. Define genus of an entire function
11. Define Convex function.
12. State Gauss's formula

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Let G be an open set which is a -star shaped. If γ_0 is the curve which is constantly equal to a then prove that every closed rectifiable curve in G is homotopic to γ_0
14. Show that $\int_0^{\pi} \frac{d\theta}{a + cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$ for $a > 1$.
15. If $|z| \leq 1|$ and $p \geq 0$ then prove that $|1 - E_p(Z)| \leq |Z|^{p+1}$
16. Let G and Ω be the regions such that there is a one to one analytic function f of G onto Ω: let $a \in G$ and $\alpha = f(a)$. If g_α and γ_α are the green's functions of G and Ω with singularities a and α respectively then prove that $g_\alpha(z) = \gamma_\alpha[f(z)]$
17. Let f be an analytic function on the disc B(a: r) such that $|f'(z) - f'(a)| < |f'(a)|$ for all z in B(a: r) , $z \neq a$ a then prove that f is one to one .

18. If G is simply connected and $f : G \to G$ is analytical in G then prove that f has a primitive in G .

19. If $G = \{z : ReZ > 0\}$ and $f_n(z) = \int_{\frac{1}{n}}^{n} e^{-t}\ t^{z-1}\ dt$ for $n \geq 1$ and z in G then prove that each f_n is analytic on G and the sequence is convergent in H(G).

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove GOURSAT' s theorem

21. State and prove SCHWARZ ‘s lemma for analytic functions

22. Let G be a region which is not a whole plane and such that every non vanishing analytic function on G has an analytic square root .If $a \in G$ G then prove that there is an analytic function f on G such that

    (i) $f(a) = 0$ and $f'(a) > 0$

    (ii) f is one to one

    (iii) $f(G) = D = z : |Z| < 1$

23. State and prove Harnack's theroem

24. State and prove Schottky's theorem