M.Sc DEGREE EXAMINATION,APRIL 2019
I Year II Semester
Differential Equations

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define analytic functions.
2. State the Legendre equation of order *p*.
3. Write down a general non-linear differential equation of order one.
4. Define fundamental matrix.
5. When is the function f(t,x) defined in a region $D \subset R^2$ said to satisfy Lipschitz condition in the variable x with a Lipschitz constant K?.
6. State Picard's theorem.
7. Write the general form of linear first order partial differential equation.
8. When do we say that the two first order partial differential equations f(x,y,z,p,q)=0 and g(x,y,z,p,q)=0 are compatible?.
9. When do you say that the operator $F(D,D^{'})$ is reducible?.
10. Define a quasi-linear PDE of second order.
11. Eliminate the arbitrary constants a and b from the equation $2z=(ax+y)^2+b$.
12. Define self adjoint operator.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that $\frac{d}{dt}\left[t^p J_p\left(t\right)\right] = \ t^p J_{p-1}\left(t\right)$.
14. Find the fundamental matrix for the system x' = Ax, where $A=\begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix}$, where $\alpha_1, \alpha_2 \; and \; \alpha_3$ are scalars.
15. Assume that f(t) and g(t) are non-negative continuous functions for $t \geq t_0$. Let $k > 0$ be a constant. Prove that the inequality $f(t) \leq k + \int_{t_0}^{t} g\left(s\right) \ f\left(s\right) ds$, $t \geq t_0$ implies the inequality $f(t) \leq k \exp\left(\int_{t_0}^{t} g\left(s\right) \ ds \right)$, $t \geq t_0$.

16. Find the general solution of the linear partial differential equation
    z(xp-yq) = $y^2 - x^2$.
17. If u = f(x+iy) + g(x-iy) where f and g are arbitrary functions, then prove that
    $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$
18. Show that the equations xp-yq=x, x^2p+q=xz are compatible.
19. Construct an operator adjoint to the Laplace operator given by
    L(u) = $u_{xx} + u_{yy}$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If P_n is a Legendre polynomial, then prove that $\int_{-1}^{1} P_n^2(t)\, dt = \frac{2}{2n+1}$.
21. Let A(t) be an n x n matrix which is continuous on I . Suppose a matrix Φ satisfies $X' = A(t)X$, $t \in I$ then prove that det Φ satisfies the first order equation $(\det \Phi)' = (\text{tr } A)(\det \Phi)$.
22. Solve the IVP $x' = x^2$, x(0) = 1 by the method of successive approximations.
23. Use Charpit's method to solve p = $(z+qy)^2$.
24. Reduce the equation $(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}$ to canonical form and find its general solution.

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