M.Sc DEGREE EXAMINATION,APRIL 2019
II Year III Semester
Differential Geometry and Tensor Calculus

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define equivalent path.
2. Define Osculating sphere.
3. Define Arc length.
4. Define direction ratios.
5. Define Helicoids.
6. State Gauss Bonnet theorem.
7. Define Geodesics.
8. Define Covariant tensor.
9. Define symmetric tensor.
10. Define Invariance.
11. State Ricci's theorem.
12. Define Metric tensor.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Calculate the curvature and torsion of the cubic curve given by r = (u,u^2,u^3).
14. Show that [r^1,r^{11},r^{111}] = 0 is a necessary and sufficient condition that the curve be plane.
15. Find a surface of a revolution which is isometric with a region of the right helicoids.
16. Prove that the curves of the family v^3/u^2 = constant are geodesics on a surface with metric v^2 du^2-2uvdudv+2$u^2$$dv^2$ (u>0, v>0)
17. Prove that if, in a mixed tensor, a contra variant of rank s and covariant of rank r, we equate a covariant and a contravariant index and sum with respect to that index, then the resulting set of n^{r+s-2} sums is a mixed tensor, covariant of rank r-1 and contravariant of ranks s-1.

18. Prove that $\frac{\partial}{\partial x^i}$ log $\sqrt{g}= \begin{Bmatrix} \alpha & \\ i & \alpha \end{Bmatrix}$

19. Prove that $\frac{\partial gij}{\partial x^k}$ $=$ [ik,j] $+$ [jk,i]

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. State and prove the fundamental existence theorem for space curves.
21. On the parabolic x^2-y^2=z find the orthogonal trajectories on the sections by the planes z=constant.
22. Prove that every helix on a cylinder is a geodesic.
23. Prove that sum of two tensors which have the same number of covariant and the same number of contravariant indices is again a tensor of the same type and rank as the given tensors.
24. Derive the transformation law for the christoffel symbol of second kind.

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18. Prove that $\frac{\partial}{\partial x^i}$ log $\sqrt{g}= \begin{Bmatrix} \alpha & \\ i & \alpha \end{Bmatrix}$

19. Prove that $\frac{\partial gij}{\partial x^k}$ = [ik,j] + [jk,i]

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