M.Sc DEGREE EXAMINATION,APRIL 2019
I Year I Semester
Real Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Lebesgue measurable set.
2. Show that there exist uncountable sets of zero measure
3. Show that if f is a non negative measurable function then $f = 0$ almost everywhere iff $\int f dx = 0$
4. Show that if f is an integrable function then $\left| \int f dx \right| \leq \int |f|\, dx$. When does equality occur?
5. Define point wise bounded sequence of functions
6. Test the convergence of $f_n(x) = \frac{sin \quad nx}{\sqrt{n}}$ for x is real, $n = 1, 2 \ldots$
7. Define Contraction of a metric space X.
8. Suppose E is a open set in R^n and f maps E into R^n. For $x \in E$ with $A = A_1$ and $A = A_2$ then prove that $A_1 = A_2$.
9. Prove that $lim_{x\to 0} \frac{log(1+x)}{x} = 1$
10. State STIRLING's formula.
11. Define orthonormal system of functions.
12. Define the integral of a simple measurable function ϕ .Also Give an example.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that the class M is a σ algebra.
14. Let f be a bounded function defined on the finite interval [a,b] then prove that f is Riemann integrable over [a,b] iff it is continuous
15. Suppose $\{f_n\}$ is a sequence of functions differentiable on [a,b] and such that $\{f_n(x_0)\}$ converges for some point x_0 on [a,b]. If f_n converges uniformly on [a,b] then prove that f'_n is uniformly continuous on [a,b] to a function f .

16. Suppose f maps a open set $E \subset R^n$ into R^m then prove that $f \in \zeta'(E)$ iff the partial derivatives $D_i f_i$ exist and are continuous on E for $1 \leq i \leq m, 1 \leq j \leq n$

17. If f is a positive function on (0, ∞) such that i) $f(x+1) = xf(x)$; ii.) $f(1) = 1$; iii.) log$f$ is convex then prove that $f(x) = \Gamma x$

18. Let $\{E_i\}$ be a sequence of measurable sets. Then

 i. If $E_1 \subseteq E_2 \subseteq ...$, then prove $m(lim\ E_i) = lim\ m(E_i)$.

 ii. If $E_1 \supseteq E_2 \supseteq ...$ and $m(E_i) < \infty$ for each i, then prove that $m(lim\ E_i) = lim\ m(E_i)$.

19. If f is continuous with period 2π andprove that there is a trigonometric polynomial P such that $|P(x) - f(x)| <\in$ for all real x.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the outer measure of an interval equals its length.

21. State and prove FATOU's lemma on non negative measurable functions.

22. If f is a continuous complex function on [a,b] prove that there exist a sequence of polynomials P_n such that $lim_{n\to\infty} P_n(x) = f(x)$ uniformly on [a,b]. If $f(x)$ is real then prove that P_n may be taken real.

23. If X is a complex metric space and if φ is a contraction of X into X then prove that there exist one and only one x $\in$ X such that φ(x) = x.

24. State and prove PARSEVAL's theorem on Riemann integrable functions.

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