

**M.Sc DEGREE EXAMINATION, APRIL 2019**  
**I Year II Semester**  
**Topology**

**Time : 3 Hours**

**Max.marks :75**

**Section A** ( $10 \times 2 = 20$ ) Marks

Answer any **TEN** questions

1. Define a topology on a nonempty set  $X$ .
2. Define nowhere dense subset of a metric space .
3. Define open base for a point.
4. Define second countable space.
5. Define Lebesgue number.
6. State Bolzano-Weierstrass property.
7. Define a sequentially compact metric space.
8. Define Hausdorff space.
9. State the Tychonoff's theorem.
10. Define a completely regular space.
11. When is a topological space disconnected ?
12. Give an example of a connected space.

**Section B** ( $5 \times 5 = 25$ ) Marks

Answer any **FIVE** questions

13. Prove that the intersection of two topologies on  $X$  is again a topology on  $X$ .
14. Let  $X$  be a topological space and  $A$  be a subset of  $X$ . Prove that  $\overline{A} = \{ x: \text{each neighbourhood of } x \text{ intersects } A \}$ .
15. Let  $X$  be a second countable space. Prove that any open base for  $X$  has a countable subclass which is also an open base.
16. Prove that any continuous image of a compact set is compact.
17. Prove that every separable metric space is second countable.
18. Prove that the product of any non empty class of Hausdorff spaces is Hausdorff.
19. Prove that any continuous image of a connected space is connected.

**Section C** ( $3 \times 10 = 30$ ) Marks

Answer any **THREE** questions

20. Let  $X$  and  $Y$  be metric space and  $f$  a mapping of  $X$  into  $Y$ . Prove that  $f$  is continuous if and only if  $f^{-1}(G)$  is open in  $X$  whenever  $G$  is open in  $Y$ .
21. State and prove Lebesgue covering Lemma.
22. Prove that the metric space is sequentially compact if and only if it has the Bolzano-Weierstrass property.
23. State and prove Urysohn's Lemma.
24. Prove that the product of any non-empty class of connected space is connected.

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