M.Sc DEGREE EXAMINATION,APRIL 2019
I Year I Semester
Probability and Distributions

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define hypergeometric random variable.
2. Define uniform distribution.
3. Define joint probability mass function of (X,Y).
4. When will you say that the random variables X and Y are identically distributed?.
5. Define bivariate normal distribution.
6. What is the joint probability mass function of X_1 and X_2 , if $X_1 = Y_1 + Y_{12}$ and $X_2 = Y_2 + y_{12}$.?.
7. Define F-distribution for the two independent χ^2 RV X and Y with m and n d.f. respectively.
8. State the t-distribution with n d.f. for the two independent variable X and Y.
9. Let $\{X_n\}$ be a sequence of RVs defined on some probability space (Ω,S,P). When will you say that the sequence $\{X_n\}$ converges in probability to the RV X?
10. State Lindberg-Levy Central Limit theorem.
11. Define Gamma distribution.
12. Define distribution function of the RV (X,Y).

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the M.G.F. of gamma distribution.
14. An Urn contains three red and two green balls. A random sample of two balls is drawn (a) with replacement (b) without replacement .

    Given that X = 0 if the first ball drawn is green, =1 if the first ball drawn is red, and Y = 0 if the second ball drawn is green , =1 if the second ball drawn is red. Find the joint PMF of (X,Y).
15. Find the joint moment generating function and covariance between X_1 and X_2 for Bivariate poisson distribution.

16. Prove that the distribution of $\sqrt{n}\ (\overline{X} - \mu)/S$ is t(n-1).
17. Let $\{X_n\}$ be a sequence of RVs such that Xn $\xrightarrow{2}$ X. Prove that E $X_n \rightarrow$ E X and E X_n^2 $\rightarrow$E X^2 as n $\rightarrow\infty$.
18. Let X $\sim$ *N* (3,4). Find P[2 <X $\leq$5).
19. Let X_1 , X_2 be independent RVs with common density given by f(x) $=\begin{cases} 1\ if\ 0 < x < 1 \\ 0\ \ otherwise. \end{cases}$

    and if $Y_1 = X_1 + X_2$and $Y_2 = X_1$ - X_2 then find the Jacobian of transformation.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If X be distributed with PDF f(x) = $\begin{cases} \frac{1}{12}x^2\,(1-x)\,,\ 0\ < x < 1 \\ 0, \qquad\qquad otherwise \end{cases}$

    and X $\sim$ B(3,2), find (i) E X (ii) E X^n (iii) var(X) (iv) M(t).
21. Let (X,Y) be jointly distributed with density function f(x,y) $= \begin{cases} x+y\ ,\ \ 0\ < x < 1,\ 0\ < y < 1 \\ 0, \qquad\qquad otherwise \end{cases}$ then find E(X^l Y^m) and cov(X,Y).
22. Compute the M.G.F. M(t_1, t_2) of a bivariate normal RV(X,Y).
23. Find the mean and variance of $\chi^2(n)$.
24. Let $X_n \xrightarrow{P}$ X and g be a continuous function defined on R.

    Prove that g (X_n) $\xrightarrow{P}$ g(X) as n$\rightarrow\infty$.

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