DEGREE EXAMINATION,APRIL 2019
I Year I Semester
Differential Calculus

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Find the n^{th} differential coefficient of $\log(ax+b)$
2. Find the n^{th} derivative of e^{ax}.
3. If $x = u(1+v)$ and $y = v(1+u)$ find $\frac{\partial(x,y)}{\partial(u,v)}$.
4. State the necessary conditions for the existence of maxima or minima at a point.
5. Write the Cartesian formula for the radius of curvature
6. Find the radius of curvature for the curves $y = e^x$ at the point where it crosses the y-axes.
7. Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point $(2,1)$
8. Find the radius of curvature of the curve $r^2 = a^2 \sin 2\theta$.
9. Define Asymptotes.
10. Show that the asymptotes of $x^2y^2 = c^2(x^2+y^2)$ are the sides of a square.
11. Find the $(p-r)$ equation of the cardioid $r = a(1-\cos\theta\ )$
12. What is the radius of curvature of the curve $x^4+y^4 = 2$ at the point $(1,1)$?

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the n^{th} derivative of $\sin 2x\ \sin 4x\ \sin 6x$
14. Find the maximum or minimum values of $2\left(x^2-y^2\right) - x^4 + y^4$
15. Show that the radius of curvature at any point of the catenary $y = c\cosh\dfrac{x}{c}$ is equal to the length of the portion of the normal intercepted between the curve and the axis of x.
16. From the polar equation of the parabola, show that $\rho^2 = ar$.
17. Find the asymptotes of $(x+y)^2\,(x+2y+2) = x+9y-2$.
18. If $y_1 = e^{a\sin^{-1}x}$ prove that $\left(1-x^2\right)y_{n+2} - (2n+1)\,xy_{n+1} - \left(n^2+a^2\right)y_n = 0$.

19. Show that the evolute of the cycloid $x = a(\theta - \sin\theta); y = a(1 - \cos\theta)$ is another cycloid.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $y = \sin\left(m\ sin^{-1}x\right)$ prove that $\left(1 - x^2\right) y_2 - xy_1 + m^2 y = 0$ and $\left(1 - x^2\right) y_{n+2} - (2n + 1)\, xy_{n+1} + \left(m^2 - n^2\right) y_n = 0$.

21. If $u = a^3x^2 + b^3y^2 + c^3z^2$ where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of u.

22. Find the radius of curvature at $x = y = \frac{3a}{2}$ to the curve $x^3 + y^3 = 3axy$

23. If $y = \left(x + \sqrt{1 + x^2}\right)^m$, prove that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0.$

24. Determine the asymptotes of the curve $4\left(x^4 + y^4\right) - 17x^2y^2 - 4x\left(4y^2 - x^2\right) + 2\left(x^2 - 2\right) = 0$ and show that they pass through the points of intersection of the curve with the ellipse $x^2 + 4y^2 = 4$.

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