**B.Sc DEGREE EXAMINATION,APRIL 2019**
**I Year II Semester**
**Integral Calculus and Fourier Series**

**Time : 3 Hours** **Max.marks :75**

**Section A** $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Evaluate $\int x^2 \ e^{-2x} \ dx$.
2. Integrate $\displaystyle\int_0^{\pi/2} \sin^9 x$ dx
3. Evaluate $\displaystyle\int \tan^3 x \ dx$
4. Evaluate $\displaystyle\int_0^1 \int_0^2 xy^2 dy \ dx$
5. Evaluate $\displaystyle\int_0^a \int_0^b \int_0^c xyz \ dz \ dy \ dx$
6. Define gamma function.
7. Prove that : $\beta\,(m,n) = \ \beta(n,m)$
8. Show that: $\acute{\Gamma}(n+1) = n\acute{\Gamma}(n)$
9. Define a Fourier series in the interval [0 , $2\pi$].
10. Find the constant $a_0$ of the Fourier series for the function:

    f(x) = x cosx in -$\pi$ < x < $\pi$
11. Without evaluating any Integral, write the half range series with Sine series for f(x) = $\sin^3$x in (0,$\pi$)
12. What is the co efficient of $b_n$ for a half range Sine series in 0 < x < l.

**Section B** $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Obtain the reduction formula for $\displaystyle\int x^m \ (log \ x)^n \ dx$.
14. Evaluate $\displaystyle\iint xy \ dy \ dx$ taken over the positive quadrant of the circles $x^2 + y^2 = a^2$

15. By Change of Order of Integration, evaluate $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dx\, dy$
16. P.T $\beta$(m,n) =2 $\int_0^{\frac{\pi}{2}} \sin^{2m-1}x \, \cos^{2n-1}x \, dx$
17. Evaluate $\int_0^{\infty} e^{-x^2} dx.$
18. Expand f(x) $=$ x (-$\pi$ $<$ x $<$ $\pi$) as a Fourier series with period $2\pi$
19. Obtain the half range Sine series for $f(x) = 1 - x$ in (0,1)

**Section C** $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Evaluate $\int_0^{\pi/2} \sin^m x \quad cos^n$ x dx , by reducing m.
21. Evaluate $\int\int\int xyz\ dx\ dy\ dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=1$
22. Prove that: $\beta\,(m,n) = \frac{\acute{\Gamma}\,(\mathrm{m})\ \acute{\Gamma}(\mathrm{n})}{\acute{\Gamma}(\mathrm{m+n})}$
23. If f(x) $= \begin{cases} -x & in \quad -\pi < x < 0 \\ x & in \quad 0 \le x < \pi \end{cases}$

    Expand f(x) as Fourier series in the interval -$\pi$ $to$ $\pi$.

    Deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$
24. Find a fourier series with period 3 to represent f(x)= 2x $-x^3$ in the range (0,3).

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