

**B.Sc DEGREE EXAMINATION, APRIL 2019**  
**II Year III Semester**  
**Allied Mathematics- I**

**Time : 3 Hours****Max.marks :75****Section A** ( $10 \times 2 = 20$ ) MarksAnswer any **TEN** questions

1. Prove that :  $\frac{e^2 - 1}{e^2 + 1} = \frac{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots}{1 + \frac{1}{2!} + \frac{1}{4!} + \dots}$
2. Prove that:  $\frac{a-x}{a} + \frac{1}{2} \left\{ \frac{a-x}{a} \right\}^2 + \frac{1}{3} \left\{ \frac{a-x}{a} \right\}^3 + \dots \infty = \log a - \log x$
3. Prove that  $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots \dots$
4. Define Hermitian matrix.
5. State Cayley Hamilton theorem.
6. Give an example of a symmetric matrix.
7. Express  $\cos 5\theta$  in terms of  $\cos \theta$  and its powers.
8. Write the expansion of  $\cos^n \theta$ .
9. Define interpolation.
10. Write Newton's backward interpolation formula.
11. If  $\tanh \frac{x}{2} = \tan \frac{\theta}{2}$ , then show that  $x = \log \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$
12. Evaluate  $\lim_{x \rightarrow 0} \frac{\sinh x - \sin x}{x^3}$

**Section B** ( $5 \times 5 = 25$ ) MarksAnswer any **FIVE** questions

13. Prove that:  $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots \infty = \frac{1}{e}$
14. Prove that:  $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$  is orthogonal.
15. Prove that:  $\frac{\sin 5\theta}{\sin \theta} = 16 \sin^4 \theta - 20 \sin^2 \theta + 5$ .

16. From the following table, estimate the value of  $y$  when  $x=21$ .

x:	20	23	26	29
y:	0.34	0.39	0.44	0.48

17. Show that  $\tanh^{-1}x = \frac{1}{2}\log\left\{\frac{1+x}{1-x}\right\}$

18. If  $\sin(A + iB) = x + iy$  prove that

$$(i) \frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1; \quad (ii) \frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$$

19. Find the eigenvalues and eigenvectors of the matrix  $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$ .

### Section C ( $3 \times 10 = 30$ ) Marks

Answer any **THREE** questions

20. Sum to infinity of the series  $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$

21. Verify Cayley - Hamilton theorem for  $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

22. Expand  $\sin^4 \theta \cos^2 \theta$  in a series of cosines of multiples of  $\theta$ .

23. Apply Newton's backward difference formula to find a polynomial of degree 3, using the table given below:

X:	3	4	5	6
Y:	6	24	60	120

24. Separate in to real and imaginary parts  $\tan^{-1}(x + iy)$ .

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