B.Sc DEGREE EXAMINATION,APRIL 2019
I Year I Semester
Allied Mathematics - I

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Sum the series $\log_3 e - \log_9 e + \log_{27} e - \ldots$
2. Give the expansion of $(1-x)^{-2}$.
3. If $y = \log(ax+b)$ find y_n.
4. If $y=\sin^{-1}x$ prove that $(1-x^2)\, y_2 - xy_1 = 0$.
5. Find the Jacobian of the transformation $x = r\cos\theta$, $y = r\sin\theta$.
6. Define the Jacobian.
7. Prove that $16 \sin^5\theta = \sin 5\theta - 5 \sin 3\theta + 10 \sin \theta$.
8. If $\frac{sin\theta}{\theta} = \frac{20165}{20166}$, then show that θ is equal to $3^\circ\ 1'$.
9. Find the integration of $x^n \log x$ with respect to x.
10. Evaluate $\int_0^{\frac{\pi}{2}} sin^7\ x\ dx$
11. Prove that $\left(\frac{a-b}{a}\right) + \frac{1}{2}\left(\frac{a-b}{a}\right)^2 + \frac{1}{3}\left(\frac{a-b}{a}\right)^3 + \cdots = log\frac{a}{b}$
12. Write down the Leibniz's formula.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.
14. Find the n^{th} differential co-efficient of cosx cos2x cos3
15. If $u = xyz$, $v = xy + yz + zx$, $w = x + y + z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$
16. If α, β, γ are the roots of the equation $x^3+px^2+qx+p=0$, prove that $tan^{-1}\alpha + tan^{-1}\beta + tan^{-1}\gamma = n\pi$.
17. If $I_n = \int sin^n x dx$ prove that $I_n = -\frac{1}{n} sin^{n-1}x cos x + \frac{n-1}{n} I_{n-2}$.

18. Prove that cos $5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.
19. If $y = \dfrac{3}{(x+1)(2x+1)}$ find y_n.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Show that $1 + \dfrac{1+3}{2!} + \dfrac{1+3+3^2}{3!} + \dfrac{1+3+3^2+3^3}{4!} + \ldots = \dfrac{e(e^2-1)}{2}$.
21. If $y = \sin(m\sin^{-1}x)$ prove that $(1-x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} + (m^2-n^2)\,y_n = 0$.
22. Find the maximum and minimum values of the function $f(x,y) = xy + \dfrac{1}{x} + \dfrac{1}{y}$.
23. Prove that cos $8\theta = 128\cos^8\theta - 256\cos^6\theta + 160\cos^4\theta - 32\cos^2\theta + 1$.
24. If $f(m,n) = \displaystyle\int_0^{\frac{\pi}{2}} cos^m\, x\; cosnx\; dx$ show that $f(m,n) = \dfrac{m}{m+n}\, f(m-1, n-1)$ and hence show that $f(n,n) = \frac{\pi}{2^{n+1}}$.

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