B.Sc. DEGREE EXAMINATION,APRIL 2019
II Year IV Semester
VECTOR CALCULUS AND FOURIER TRANSFORMS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Gradient of a scalar point function.
2. Find the unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at the point (2,0,1).
3. What is the necessary and sufficient condition for $\overrightarrow{F}$ to be a conservative force?
4. State Green's theorem.
5. If V is the volume enclosed by a closed surface and $\overrightarrow{r} = x\overrightarrow{i} + y\overrightarrow{j} + z\overrightarrow{k}$ show that $\iint_s \overrightarrow{r}.\ \overrightarrow{n}\ ds = 3V$.
6. State Stoke's theorem.
7. State the Dirichlet's condition.
8. Write down the Fourier transform and its inverse transformation in complex form.
9. Define unit step function.
10. If $F_s[f(x)] = F_s(s)$ is the Fourier sine transform of f(x) then prove that $F_s[f(ax)] = \frac{1}{a} F_s\left(\frac{s}{a}\right)$.
11. Show that grad $(\overrightarrow{r}.\ \overrightarrow{a}) = \overrightarrow{a}$.
12. Write the Fourier sine transform of f(x).

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the equation of the tangent plane and normal to the surface xyz = 4 at the point (1,2,2).
14. If $\overrightarrow{F} = yz\overrightarrow{i} + zx\overrightarrow{j} - xy\overrightarrow{k}$ find $\int_c \overrightarrow{F}.\overrightarrow{dr}$ where C is given by $x = t$, $y = t^2$, $z = t^3$ from (0,0,0,) to (2,4,8).
15. Using Stoke's theorem prove that
$$\int_c \overrightarrow{r}.d\overrightarrow{r} = 0 .$$

16. Find the Fourier transform $f\left(x\right)=\begin{cases} x, & |x|<a \\ 0, & |x|>a \end{cases}$
17. State and prove the convolution theorem.
18. Find the $\emptyset$ if $\nabla\emptyset=\left(y+\sin z\right)\overrightarrow{i}+x\overrightarrow{j}+xcosz\ \overrightarrow{k}$.
19. Find Fourier cosine transform of $e^{-x}+e^{-2x}$.

Section C $(3\times 10=30)$ Marks

Answer any **THREE** questions

20. (i) Prove that div $\overrightarrow{r}=3$ and curl $\overrightarrow{r}=0$ where $\overrightarrow{r}$ is the position vector of the point (x,y,z).

    (ii) Find the directional derivative of $xyz-xy^2z^3$ at the point (1,2,-1) in the direction of the vector $\overrightarrow{i}$ - $\overrightarrow{j}-3\overrightarrow{k}$.
21. Verify Green's theorem for $\oint_c (xy+y^2)\,dx+x^2\,dy$ where c is the closed curve of the region bounded by $y=x$ and $y=x^2$.
22. Verify Gauss divergence theorem for $\overrightarrow{F}=x\overrightarrow{i}+y\overrightarrow{j}+z\overrightarrow{k}$ taken over the region bounded by the planes x= 0 ; x = a ; y = 0 ; y = a ; z = 0 ; z = a.
23. Find the Fourier transform of $e^{-|x|}$.
24. Solve the integral equation $\int_0^\infty f\left(x\right)\sin sx\ dx \ =\begin{cases} 1 & 0\leq s<1 \\ 2 & 0\ \leq s<2 \end{cases}$

B.Sc. DEGREE EXAMINATION,APRIL 2019
II Year IV Semester
VECTOR CALCULUS AND FOURIER TRANSFORMS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define Gradient of a scalar point function.
2. Find the unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at the point (2,0,1).
3. What is the necessary and sufficient condition for $\vec{F}$ to be a conservative force?
4. State Green's theorem.
5. If V is the volume enclosed by a closed surface and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ show that $\iint_s \vec{r}.\ \vec{n}\ ds = 3V$.
6. State Stoke's theorem.
7. State the Dirichlet's condition.
8. Write down the Fourier transform and its inverse transformation in complex form.
9. Define unit step function.
10. If $F_s[f(x)] = F_s(s)$ is the Fourier sine transform of f(x) then prove that $F_s[f(ax)] = \frac{1}{a} F_s\left(\frac{s}{a}\right)$.
11. Show that grad $(\vec{r}.\ \vec{a}) = \vec{a}$.
12. Write the Fourier sine transform of f(x).

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the equation of the tangent plane and normal to the surface xyz = 4 at the point (1,2,2).
14. If $\vec{F} = yz\vec{i} + zx\vec{j} - xy\vec{k}$ find $\int_c \vec{F}.\vec{dr}$ where C is given by $x = t$, $y = t^2$, $z = t^3$ from (0,0,0,) to (2,4,8).
15. Using Stoke's theorem prove that $\int_c \vec{r}.d\vec{r} = 0$.

16. Find the Fourier transform $f(x) = \begin{cases} x, & |x| < a \\ 0, & |x| > a \end{cases}$
17. State and prove the convolution theorem.
18. Find the $\emptyset$ if $\nabla\emptyset = (y + \sin z)\ \overrightarrow{i} + x\overrightarrow{j} + xcosz\ \overrightarrow{k}$.
19. Find Fourier cosine transform of $e^{-x} + e^{-2x}$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. (i) Prove that div $\overrightarrow{r} = 3$ and curl $\overrightarrow{r} = 0$ where $\overrightarrow{r}$ is the position vector of the point (x,y,z).

    (ii) Find the directional derivative of $xyz - xy^2z^3$ at the point (1,2,-1) in the direction of the vector $\overrightarrow{i}$ - $\overrightarrow{j} - 3\overrightarrow{k}$.
21. Verify Green's theorem for $\oint_c (xy + y^2)\, dx + x^2\, dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$.
22. Verify Gauss divergence theorem for $\overrightarrow{F} = x\overrightarrow{i} + y\overrightarrow{j} + z\overrightarrow{k}$ taken over the region bounded by the planes x= 0 ; x = a ; y = 0 ; y = a ; z = 0 ; z = a.
23. Find the Fourier transform of $e^{-|x|}$.
24. Solve the integral equation $\int_0^\infty f(x) \sin sx\ dx = \begin{cases} 1 & 0 \le s < 1 \\ 2 & 0 \le s < 2 \end{cases}$