B.Sc DEGREE EXAMINATION,APRIL 2019
III Year V Semester
Modern Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. If G is finite group and a$\in$G , prove that a$^{o(G)}$ $=$ e.
2. Show that every subgroup H of G of index 2 is normal subgroup of G.
3. Let G be the group of integers under addition and let φ: G $\rightarrow$ G be defined by φ(x)=2x. Show that φ is homomorphism.
4. Define an automorphism.
5. Find the cycles of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$
6. If R is any ring, show that a(-b)=(-a)b=-(a b), for all a, b $\in$ R.
7. Define a maximal ideal of a ring R.
8. Let φ: R$\rightarrow$R^1 be a homomorphism of rings. Show that if φ is an isomorphism then I(φ)=0.
9. Define a Euclidean ring.
10. Show that $1 +$ i is a prime element of J[i].
11. When do you say a polynomial is integer monic.
12. When do you say an integral domain R with unit element is a unique factorisation domain.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Let G be a group and let H, K be two subgroups of G. Prove that HK is a subgroup of G if and only if HK $=$ KH.
14. Let G be a group and N be a subgroup of G. Prove that N is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.
15. Let G be group. Show that the set A(G) of all automorphisms of G is a group.
16. Show that every permutation can be expressed as a product of disjoint cycles.
17. Let G be an ideal of the ring R. Show that R/U is a ring.

18. Let R be a Euclidean ring and let a, b $\in$ R. Define the greatest common divisor *d* of *a* and *b* and show that $d = \lambda a + \mu b$ for some λ, $\mu \in R$.
19. Let $f(x)$ and $g(x)$ be primitive polynomials. Prove that $f(x)g(x)$ is a primitive polynomial.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If H and K are finite subgroups of G of orders O(H) and O(K) respectively. Prove that O(HK) = O(H)O(K)/O(H$\cap$K)
21. Let φ be a homomorphism of G onto $\overline{G}$with kernel K. Prove that G/ K $\cong \overline{G}$.
22. Show that a commutative ring R with unit element $1 \neq 0$ is a field if and only if (0) and R are the only ideals of R.
23. Show that every integral domain can be imbedded in a field.
24. If *R* is a unique factorisation domain and if *p*(*x*) is a primitive polynomial in *R*[*x*]. Prove that it can be factored in a unique way as the product of irreducible element in *R*[*x*].

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