## B.Sc DEGREE EXAMINATION,APRIL 2019
## III Year V Semester
## Real Analysis

**Time : 3 Hours** **Max.marks :75**

**Section A** $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define a countable set with an example .
2. State Least Upper bound axiom.
3. Define a bounded sequence.
4. Prove that $\sum_{n=1}^{8} \frac{1}{n(n+1)}$ converges.
5. If a sequence $\{s_n\}_{n=1}^{8}$ converges to 0 and $s_n > 0$ for $n \in N$, then prove that the sequence $\left\{\frac{1}{S_n}\right\}_{n=1}^{8}$ diverges to $\infty$.
6. If f is continuous at $a \in R'$ then prove that $|f|$ is also continuous at a.
7. Define an open ball in a metric space.
8. Define a connected set.
9. Define a complete metric space.
10. Prove that if the real-valued function f has a derivative at the point $c \in R'$, then f is continuous at c.
11. If $f'(x) = 0$ for every $x$ in the closed bounded interval [a,b] then prove that f is constant on [a,b].
12. State Rolle's theorem.

**Section B** $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $A_1,\ A_2,\ \ldots$ are countable sets, then prove that $\bigcup_{n=1}^{\infty} A_n$ is countable.
14. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.
15. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges, then prove that $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

16. Prove that if $\sum_{n=1}^{\infty} a_n$ is a convergent series then $lim_{n\to\infty}\ a_n = 0.$
17. If $G_1$ and $G_2$ are open subsets of the metric space M then prove that $G_1 \cap G_2$ is also open.
18. Let f be a continuous function from a metric space $M_1$ into a metric space $M_2$. If $M_1$ is connected, then prove that the range of f is also connected.
19. State and prove the Darboux property.

**Section C** $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Define a non-decreasing sequence and prove that a nondecreasing sequence which is bounded above is convergent.
21. If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that

    (a.) $a_1 \geq a_2 \geq \cdots \geq a_n \geq a_{n+1} \geq \ldots$ and

    (b.) $lim_{n\to\infty} a_n = 0$, then prove that the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.
22. Prove that

    (i) if G is an open subset of the metric space M, then $G' = M - G$ is closed.

    (ii) if F is closed subset of M, then $F' = M - F$ is open.

23.. Let $(M, \rho)$ be a complete metric space. Prove that if T is a contraction on M, then there is one and only one point x in M such that Tx=x.

24. State and prove the second Fundamental theorem of calculus.

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