B.Sc DEGREE EXAMINATION,APRIL 2019
I Year II Semester
Classical Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. State Binomial theorem
2. Find the coefficient of x^n in the expansion of e^{a+bx}
3. Give an example for symmetric and skew symmetric matrices.
4. Prove that the matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is unitary.
5. Form a rational cubic equation which shall have for roots $1,\ 3-\sqrt{-2}$.
6. Frame an equation with rational coefficients, one of whose roots is $\sqrt{5}+\sqrt{2}$
7. If $\alpha,\ \beta,\ \gamma$ are the roots of the equation $ax^3+bx^2+cx+d=0$, find the equation whose roots are $\alpha^2,\ \beta^2,\ \gamma^2$
8. If $\alpha,\ \beta,\ \gamma$ are the roots of the equation $x^3+qx+r=0$, find the equation whose roots are $\beta+\ \gamma-2\alpha,\quad \gamma+\alpha-2\beta,\quad \alpha+\beta-2\gamma$.
9. Find the number and sum of all the divisors of 360
10. Find the number of integers less than n and prime to it when $n = 729$ and 720.
11. State Wilson's theorem.
12. Show that $\begin{bmatrix} 4 & 1-i \\ 1+i & 2 \end{bmatrix}$ is Hermitian.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the sum to infinity of the series $1+\frac{2^3}{2!}+\frac{3^3}{3!}+\frac{4^3}{4!}+\ldots\infty$
14. Find the eigen value and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$
15. Solve the equation $x^3-12x^2+39x-28=0$ whose roots are in A.P.
16. If $\alpha,\ \beta,\ \gamma$ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $(\alpha^2+1)(\beta^2+1)(\gamma^2+1)$.

17. Find a number having the remainders $5,\ 4,\ 3,\ 2$ when divided by $6,\ 5,\ 4,\ 3$ respectively.

18. Show that every square matrix can be uniquely expressed as the sum of a symmetric and skew-symmetric matrix.

19. If $\log\left(1-x+x^2\right) = a_1x+a_2x^2+a_3x^3+\ldots\infty$ show that $a_3+a_6+a_9 = \dfrac{3}{2}\log 2$.

Section C $(3\times 10 = 30)$ Marks

Answer any **THREE** questions

20. Find sum to infinity of the series $\dfrac{1\ .\ 3}{2\ .\ 4\ .\ 6\ .\ 8} + \dfrac{1\ .\ 3\ .\ 5}{2\ .\ 4\ .\ 6\ .\ 8\ .\ 10} + \dfrac{1\ .\ 3\ .\ 5\ .\ 7}{2\ .\ 4\ .\ 6\ .\ 8\ .\ 10\ .\ 12} + \ldots\infty$

21. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Also find A^{-1}

22. If the sum of two roots of the equation $x^4+px^3+qx^2+rx+s=0$ equals the sum of the other two, prove that $p^3+8r=4pq$.

23. Solve the equation $6x^6-35x^5+56x^4-56x^2+35x-6=0$.

24. Show that if x and y are both prime to the prime number n, then prove that $x^{n-1}-y^{n-1}$ is divisible by n. Deduce that $x^{12}-y^{12}$ is divisible by 1365.

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9. Find the number and sum of all the divisors of 360
10. Find the number of integers less than n and prime to it when $n=729$ and 720.
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