

B.Sc DEGREE EXAMINATION, APRIL 2019
I Year II Semester
Integral Calculus and Fourier Series

Time : 3 Hours

Max.marks :75

Section A ($10 \times 2 = 20$) Marks

Answer any **TEN** questions

1. State Bernoulli's formula for integration.
2. Evaluate $\int x^4 \sin\left(\frac{x}{2}\right) dx$.
3. Evaluate $\int_0^1 \int_0^2 (x^2 + y^2) dy dx$
4. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\cos \theta} \int_0^{\sqrt{a^2 - x^2}} dz dr d\theta$
5. Define Beta function
6. write down the relation between beta and gamma function.
7. State the fourier coefficients in fourier series in $[0, 2\pi]$.
8. State Dirichlet Conditions.
9. Define Half range cosine series in $(0, l)$.
10. Find a_0 for the function $f(x) = x$ for $0 < x < \frac{\pi}{2}$
11. Evaluate $\int x^4 e^{2x} dx$
12. Prove that $\beta(m, n) = \beta(n, m)$.

Section B ($5 \times 5 = 25$) Marks

Answer any **FIVE** questions

13. Evaluate $\int_0^{\frac{\pi}{2}} x^n \sin x dx$ by using reduction formula
14. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dx dy dz$.
15. Prove that $\Gamma \frac{1}{2} = \sqrt{\pi}$
16. Obtain a Fourier expansion for the series

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$$

17. Obtain a sine series for unity in $0 < x < \pi$

18. Prove that $\frac{\beta(m+1, n)}{m} = \frac{\beta(m, n)}{m+n}$

19. Evaluate $\int (x^3 \log x)^2 dx$

Section C ($3 \times 10 = 30$) Marks

Answer any **THREE** questions

20. If $I_n = \int_0^{\frac{\pi}{2}} x^n \cos x dx$ Show that $I_n + n(n-1) I_{n-2} = \left(\frac{\pi}{2}\right)^n$ for $n > 1$.

21. Change the order of integration and hence evaluate $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}} dx dy$

22. Show that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$

23. Find the fourier series expansion of $x \sin x$ in the interval $-\pi \leq x \leq \pi$.

24. Find the Half range sine series of $f(x) = x \cos x$ in $(0, \pi)$.

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