B.Sc. DEGREE EXAMINATION,APRIL 2019
II Year IV Semester
VECTOR CALCULUS, FOURIER TRANSFORMS AND Z TRANSFORMS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Show that the vector $\overline{\mathbf{F}}=\mathbf{3y^4z^2\bar{i}+4x^3z^2\bar{j}-3x^2y^2\overline{k}}$ is solenoidal.
2. Define an irrotational vector.
3. State Green's theorem in the plane.
4. Define line integral.
5. State Gauss divergence theorem.
6. State Stoke's theorem.
7. Write down the Fourier sine transform of $\mathbf{f(x)}$.
8. Define the convolution of two functions in Fourier transform.
9. Define Z-transform of a sequence.
10. Find the Z-transform of $\mathbf{a^{n+3}}$.
11. If $\phi=\mathbf{3x^2y}z$, find $\mathbf{g}rad\ \phi$ at the point $(\mathbf{1},-\mathbf{2},-\mathbf{1})$.
12. Find the unit vector in the direction of the vector $\mathbf{\bar{i}+2\bar{j}+2\overline{k}}$.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the directional derivative of $\mathbf{x^2+y^2+4y}zx$ at $(\mathbf{1},-\mathbf{2},\mathbf{2})$ in the direction $\mathbf{2\bar{i}-2\bar{j}+\overline{k}}$.
14. If $\overline{\mathbf{A}}=\left(\mathbf{3x^2+6y}\right)\mathbf{\bar{i}+14y}z\mathbf{\bar{j}+20xz^2\overline{k}}$, evaluate $\int_{\mathbf{C}}\overline{\mathbf{A}}.\mathbf{d\bar{r}}$ from $(\mathbf{0},\mathbf{0},\mathbf{0})$ to $(\mathbf{1},\mathbf{1},\mathbf{1})$ over the curve $\mathbf{x=t},\mathbf{y=t^2},\mathbf{z=t^3}$ and $\mathbf{\bar{r}}$ is the position vector.
15. If $\overline{\mathbf{F}}=\mathbf{c}curl\ \overline{\mathbf{A}}$, prove that $\int_{\mathbf{S}}\int\overline{\mathbf{F}}.\mathbf{\bar{n}d}S=\mathbf{0}$ for any closed surface S.
16. Prove that $\mathbf{F}\left[\mathbf{f}\left(\mathbf{x-a}\right)\right]=\mathbf{e}^{\mathbf{i}as}\mathbf{F(s)}$.
17. If $\mathbf{Z}\left(\mathbf{u_n}\right)=\mathbf{\bar{u}(z)}$, then prove that $\mathbf{Z}\left(\mathbf{\bar{a}^n u_n}\right)=\mathbf{\bar{u}}(\mathbf{a}z)$ and $\mathbf{Z}\left(\mathbf{a^n u_n}\right)=\mathbf{\bar{u}}(\frac{\mathbf{z}}{\mathbf{a}})$.
18. Find the unit vector normal to $\phi=\mathbf{x^2-y^2+z^2}$ at the point $(\mathbf{1},-\mathbf{1},\mathbf{2})$.

19. Evaluate $\int_{\mathbf{0}}^{\mathbf{a}} \int_{\mathbf{0}}^{\mathbf{a}} \int_{\mathbf{0}}^{\mathbf{a}} (\mathbf{4z}-\mathbf{y})\, \mathbf{d}x dy dz$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $\phi = \mathbf{x^3}+\mathbf{y^3}+\mathbf{z^3}-\mathbf{3}\mathbf{x}yz$, find $\mathbf{d}iv\ \mathbf{g}rad\ \phi$ and $\mathbf{c}url\ \mathbf{g}rad\ \phi$.
21. Evaluate using Green's theorem in the plane for $\int_{\mathbf{C}} (\mathbf{x}y+\mathbf{y^2})\, \mathbf{d}x+\mathbf{x^2}\mathbf{d}y$ where C is the closed curve of the region bounded by $\mathbf{y}=\mathbf{x}$ and $\mathbf{y}=\mathbf{x^2}$.
22. Using Gauss divergence theorem, evaluate $\int_{\mathbf{S}} \int \mathbf{x}dydz+\mathbf{y}dzdx+\mathbf{z}dxdy$ over the surface of the sphere $\mathbf{x^2}+\mathbf{y^2}+\mathbf{z^2}=\mathbf{a^2}$.
23. Using Parseval's theorem, evaluate $\int_{\mathbf{0}}^{\infty} \frac{\mathbf{x^2}}{(\mathbf{x^2}+\mathbf{a^2})^{\mathbf{2}}} \mathbf{d}x, \mathbf{a}>\mathbf{0}$.
24. Find the Z-transform of $\mathbf{c}os\ \mathbf{n}\theta$ and $\mathbf{s}in\ \mathbf{n}\theta$.

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