**B.Sc. DEGREE EXAMINATION,APRIL 2019**
**III Year V Semester**
**MODERN ALGEBRA**

**Time : 3 Hours** **Max.marks :75**

**Section A** $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Prove that the identity element of a group is unique.
2. Define a subgroup of a group.
3. Define a normal subgroup of a group G .
4. List out all the elements of $S_3$ .
5. Define even permutation.
6. Express inverse of the cycle (1 2 4 5 3) as a product of transpositions.
7. Define zero divisor.
8. State the Pigeonhole principle.
9. If U is an ideal of R and 1$\in$ U then prove that U $=$ R.
10. Define a maximal ideal of a ring R .
11. Define an Euclidean ring.
12. Let R be an integral domain with unit element. Let a, such that both b $\in$ R a / b and b / a . What best can you say about a and b ?

**Section B** $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that if H , K are subgroups of a group G then H$\cap$K is also a subgroup of G .
14. Prove that N is a normal subgroup of G if and only if gNg $^{-1}$ $=$ N for all g $\in$G.
15. Prove that the Kernel of a homomorphism $\phi$ on a group G is a normal subgroup of G.
16. If $\theta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$ is a the permutation, write the orbit of 1 and hence write all the cycles of $\theta$.
17. For any prime number p , prove that the ring of integers mod p, $J_p$ is a field.
18. Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Prove that R is a field.

19. Let R be a Euclidean ring and a,b $\in$ R . If b $\neq$ 0 is not a unit in R then prove that d(a) $<$ d(ab).

**Section C** $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If H and K are subgroups of G, then prove that $o\,(HK)= \dfrac{o\,(H)\;\;o(K)}{o(H\;\;\cap\;\;K)}$.
21. State and prove Cayley's theorem.
22. Show that a finite integral domain is a field.
23. If U,V are ideals of a ring R ,then prove that U+V and UV are ideals in R.
24. Prove that every integral domain can be imbedded in a field.

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