B.Sc DEGREE EXAMINATION,APRIL 2019
III Year V Semester
Graph Theory

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define isomorphism of graphs.
2. Define cut vertex and give an example.
3. Define diameter of a connected graph G.
4. Give an example of graph which is Eulerian but not Hamiltonian.
5. What is Chinese postman problem?
6. Define bipartite graph.
7. Define adjacency matrix of a graph G.
8. If G is a plane (p, q) graph with p = 3 , show that q = 3p – 6.
9. Define dual of a plane graph.
10. Define edge contraction of graph G.
11. Prove that if H is a subgraph of a graph G, then χ(G) = χ(H).
12. Define k-edge colouring of a graph G.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Prove that if q = p - 1, then any (p, q) graph is either connected or contains a cycle.
14. Prove that if W is a walk connecting u and v in a graph G, then W contains a path connecting u and v.
15. Prove that if G is a Hamiltonian graph, then w(G – S)= |S|, for every non-empty subset S of V(G).
16. Prove that every connected graph G contains a spanning tree.
17. Prove that if G is a connected graph, then the distance between v_i and $v_j$is the smallest integer n (= 0) such that $[A^n]_{ij} \neq 0$.
18. Prove that if G is plane (p, q) graph with δ (G)$\geq$ 3, then there is a face in G of degree $\leq$ 5.

19. Let G be a graph and let u and v be non-adjacent vertices in G, prove that χ(G) = min {χ (G+(u, v)), χ (G, uv)}.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that for any graph q(G) = p(G) – w(G).
21. Prove that if G is a (p, q) – graph (p = 3) such that deg(u)+deg(v) = p for every pair u, v of non-adjacent vertices in G then G is Hamiltonian graph.
22. Prove that a (p, q) – graph G is bipartite if and only if it contains no odd cycles.
23. State and prove Euler formula for planar graphs.
24. Prove that for any given integer k (= 1) there exists a triangle free graph with chromatic number k.

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