B.Sc DEGREE EXAMINATION,APRIL 2019
III Year VI Semester
Linear Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define basis of a vector space.
2. If V is a vector space over F, then prove that $(-\alpha)v = -(\alpha)v$ for $\alpha \in F, v \in V$
3. Define linear combination of vector.
4. Define annihilator of W, where W is a subspace of vector space V
5. If $\dim_F V = m$,then find $\dim_F Hom(V, F)$
6. Prove that $W^{\perp}$ is a subspace of V.
7. Define a orthonormal set of vectors.
8. When a linear transformation is said to be regular?
9. Define characteristic vector of a Linear transformation
10. When two linear transformations are said to be similar?
11. If $T \in A(V)$, define the range of T.
12. Prove that L(S) is a subspace of V.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Show that the vectors (0,1,1), (1,0,1) and (1,1,0) form a basis of $V_3(R)$.
14. If V is a finite dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in \hat{v}$ such that $f(v) \neq 0$
15. State and prove Schwarz inequality of norm.
16. Let V be finite-dimensional over F, prove that $T \in A(V)$is singular if and only if there exists $v \neq 0 \in V$ such that $vT = 0$
17. Find the characteristic value and characteristic vector of
    A= $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$
18. If $T, S \in A(V)$ and if S is regular, then prove that T and STS^{-1} have the same minimal polynomial.

19. If V is a finite – dimensional and W is a subspace of V, prove that $A(A(W)) = W$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $v_1, v_2, ...v_n$ are vectors in V, then prove that either they are linearly independent or some $v_k$is a linear combination of the preceding vectors $v_1, v_2, ...v_{k-1}$
21. If V is a finite dimensional vector space and if W is a subspace of V then prove that $\dim W \leq \dim V$ and $\dim \frac{V}{W} = \dim V - \dim W$
22. Prove that every finite dimensional inner product space has an orthonormal basis
23. If A is an algebra with unit element over F, prove that A is Isomorphic to a subalgebra of A(V) for some vector V over F,
24. If $T \in A(V)$ has all its characteristic roots in F, then prove that, there is a basis of V in which the matrix of T is triangular.

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