B.Sc DEGREE EXAMINATION,APRIL 2019
III Year VI Semester
Complex Analysis

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Show that the function $f(z) = (\overline{z})^2$is not analytic everywhere.
2. Show that the function $u = \sin x \cosh y$ is harmonic.
3. State Cauchy's Goursat theorem.
4. Show that $\int_c \frac{z}{(9-z^2)(z+i)} dz = \frac{\pi}{5}$, where c is the circle $|z| = 2$.
5. State fundamental theorem of algebra.
6. Find the Laurent series expansion of $f(z) = \frac{1}{z^2 - z - 2}$in the region $1 < |z| < 2$.
7. Find the residue of $\frac{ze^z}{(z-1)^2}$at its poles.
8. Define removable singularity.
9. Define bilinear transformation and give an example.
10. Find the fixed points of the transformation $w = \frac{z-1}{z+1}$.
11. State Cauchy's integral formula.
12. Find α so that $u(x, y) = \alpha x^2 - y^2 + xy$ is harmonic.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$, find the analytic function $f(z) = u + iv$.
14. Evaluate the integral $\int_c \frac{dz}{(z^2+1)(z^2-4)}$, where c is the circle $|z| = \frac{3}{2}$.
15. State and prove Liouville's theorem.
16. State and prove Cauchy's residue theorem.
17. Find the bilinear transformation which maps the points i, $-i$, 1 in the z plane onto the points 0, 1, ∞ respectively in the w plane.
18. State and prove Cauchy's integral formula.
19. Evaluate $\int_{|z-2|=2} \frac{3z^3+2}{(z-1)(z^2+9)} dz$.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $f(z) = u(r,\theta) + iv(r,\theta)$ is differential at $z = re^{i\theta} \neq 0$, show that $\dfrac{\partial u}{\partial r} = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}$ and $\dfrac{\partial v}{\partial r} = -\dfrac{1}{r}\dfrac{\partial u}{\partial \theta}$.
21. If $f(z)$ is analytic inside and on a simple closed curve C, show that $f'(z_0) = \dfrac{1}{2\pi i}\displaystyle\int_c \dfrac{f(z)}{(z-z_0)^2}dz$. Hence evaluate $\displaystyle\int_c \dfrac{\sin z}{\left(z - \frac{\pi}{2}\right)^2}dz$, where C is the circle $|z| = 2$.
22. State and prove Laurent's theorem.
23. Evaluate $\displaystyle\int_{|z|=\frac{3}{2}} \dfrac{3z-4}{z(z-1)(z-3)}dz\cdot$
24. Discuss the transformation $w = z^2$.

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