B.Sc DEGREE EXAMINATION,APRIL 2019
I Year I Semester
Allied Mathematics - I

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Write the expansion for $(1+x)^{-2}$ where $|x| < 1$.
2. If $n > 1$, show that $\frac{1}{n^2} + \frac{1}{2n^4} + \frac{1}{3n^6} + \cdots\infty = \log\left(\frac{n^2}{n^2-1}\right)$.
3. Define symmetric and skew-symmetric matrix.
4. Find the eigenvalues of the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.
5. Write the expansion of $\cos n\theta$ in powers of $\sin\theta$ and $\cos\theta$.
6. Prove that $\cos 4\theta = 8\sin^4\theta - 8\sin^2\theta + 1$.
7. Show that $D = \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} \cdots\right]$.
8. Prove that $(1+\Delta)(1-\nabla) = 1$.
9. Prove that $\cosh^2 x - \sinh^2 x = 1$.
10. Separate into real and imaginary parts of $\sinh(x+iy)$.
11. Verify Cayley - Hamilton for the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.
12. Obtain the divided difference table for

x	:	1	2	7	8
$f(x)$	:	1	5	5	4

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Find the sum of the series $1 - \frac{1}{4} + \frac{1\cdot 3}{4\cdot 8} - \frac{1\cdot 3\cdot 5}{4\cdot 8\cdot 12} + \cdots\cdot\infty\cdot$
14. Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix}$.

15. Prove that $\dfrac{\sin 6\theta}{\sin\theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$.

16. Using Lagrange's interpolation formula, find $y(10)$ from the following table.

x	:	5	6	9	11
y	:	12	13	14	16

17. If $\sin(\theta + i\phi) = \tan(x + iy)$, show that $\dfrac{\tan\theta}{\tanh\phi} = \dfrac{\sin 2x}{\sinh 2y}$.

18. Verify Cayley –Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$.

19. Find the value of y at $x = 21$ from the following data.

x	:	20	23	26	29
y	:	0.3420	0.3907	0.4384	0.4848

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Sum to infinity the series $\dfrac{1^2}{1!} + \dfrac{1^2 + 2^2}{2!} + \dfrac{1^2 + 2^2 + 3^2}{3!} + \cdots\infty$.

21. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

22. Prove that $2^8\sin^9\theta = \sin 9\theta - 9\sin 7\theta + 36\sin 5\theta - 84\sin 3\theta + 126\sin\theta$.

23. From the following table of half yearly premium for polices maturing at different ages, estimate the premium for polices maturing age 46 and 63.

Age	x	:	45	50	55	60	65
$\mathrm{Pr}emium$	y	:	114.84	96.16	83.32	74.48	68.48

24. If $\cos\alpha + i\sin\alpha = \cos(\theta + i\phi)$, prove that (i) $\sin^2\theta = \pm\sin\alpha$, (ii) $\cos 2\theta + \cosh 2\phi = 2$.

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