B.Sc. DEGREE EXAMINATION,NOVEMBER 2019
III Year VI Semester
Linear Algebra

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Justify : R is a vector space over C
2. State the condition for a non empty subset W of V is a subspace of a vectorspace V
3. Define annihilator A(W)
4. For what value of k, the set $\{(2,-1,3),(3,4,-1)(k,2,1)\}$ becomes linearly independent.
5. Prove that $W^{\perp}$ is a subspace of vector space V
6. Define basis of a vector space.
7. Show that $T : R^2 \rightarrow R^3$ by $T(a,b) = (2a-3b, a+4b)$ is a Linear Transformation.
8. When a Linear Transformation T is said to be regular.
9. Define Characteristic roots of a matrix.
10. When a Linear Transformation are said to be similar.
11. If $\alpha = (1,2,3,4)$ and $\beta = (2,0,-3,1)$ then find $\|\alpha+\beta\|$
12. Prove that L(S) is a subspace of V.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If V is the internal direct sum of $U_1, U_2, ...U_n$then prove that V is isomorphic to the external direct sum of $U_1, U_2, ...U_n$.
14. Prove that any 2 bases of finite dimensional vector space V have same number of elements.
15. Let V be the set of all polynomials of degree ≤ 2 over $\mathbb{R}$ with inner product defined by $\langle f,g \rangle = \int_0^1 f(x)g(x)dx$.Starting with the basis $\{1,x,x^2\}$ obtain orthonormal basis for V.

16. If $T, S \in A(V)$ and if S is regular, then prove that T and STS^{-1} have the same minimal polynomial.

17. Find the characteristic root of $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

18. Show that the following vector form a basis for R^3.

    (1,1,0),(0,1,1),(1,0,1)

19. Let V is finite dimensional and $\nu \neq 0$ Prove that there is an element $\overline{f} \in v$ such that $f(v) \neq 0$

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If V is a finite-dimensional vector space, then prove that it contains a finite set $v_1, v_2..........v_n$ of linearly independent elements whose linear span is V.

21. If A and B are finite dimensional subspaces of a vector space V, then prove that A+B is finite dimensional and $\dim(A+B) = \dim(A) + \dim(B) - \dim(A \cap B)$

22. Prove that every finite dimensional inner product space has an orthonormal basis.

23. If A is an algebra with unit element, over F, then prove that A is isomorphic to a sub algebra of A(v) for some vector space V over F.

24. If $T \in A(\nu)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

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