

B.Sc. DEGREE EXAMINATION, NOVEMBER 2019
I Year I Semester
Allied Mathematics-I

Time : 3 Hours

Max.marks :75

Section A ($10 \times 2 = 20$) MarksAnswer any **TEN** questions

1. Write the expansion of $(1 + x)^n$.
2. What is the value of $\log(1 + x)$.
3. State Leibnitz theorem.
4. If $y = \log(ax + b)$, then find y_n .
5. Write the necessary and sufficient condition for the extreme value of function of two variables.
6. If $x = r \cos \theta$, $y = r \sin \theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.
7. Write $\sin n\theta$ in terms of powers of $\cos \theta$ and $\sin \theta$
8. If $\frac{\sin \theta}{\theta} = \frac{5045}{5046}$, Show that $\theta = 1^\circ 58'$.
9. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^9 x dx$.
10. Evaluate $\int x \log x dx$.
11. Expand e^2 .
12. Evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x dx$.

Section B ($5 \times 5 = 25$) MarksAnswer any **FIVE** questions

13. Prove that $\log_3 e - \log_9 e + \log_{27} e - \dots = \frac{\log_e 2}{\log_e 3}$.
14. Find y_n , if $y = \frac{1}{x^2 + 5x + 6}$.
15. If $u = xyz$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

16. Prove that $\frac{\sin 6\theta}{\sin \theta} = 32 \cos^5 \theta - 32 \cos^3 \theta + 6 \cos \theta$.

17. If $u_n = \int_0^a x^n e^{-x} dx$, prove that

(i) $u_n = -e^{-a} a^n + n u_n$

(ii) $u_n - (n + a)u_{n-1} + a(n - 1)u_{n-2} = 0$

18. Prove that $\sum_{n=0}^{\infty} \frac{5n + 1}{(2n + 1)!} = \frac{e}{2} + \frac{2}{e}$.

19. Prove that $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x dx$, then show that $I_n = n \left(\frac{\pi}{2}\right)^{n-1} - n(n - 1) I_{n-2}$.

Section C (3 × 10 = 30) Marks

Answer any **THREE** questions

20. Show that $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \dots = 27e$.

21. If $y = \left(x + \sqrt{1 + x^2}\right)^m$ prove that $(1 + x^2) y_{n+2} + (2n + 1) x y_{n+1} + (n^2 - m^2) y_n = 0$.

22. Find the maximum and minimum values of the function
 $f(x, y) = x^2 y^2 - x^2 - y^2$.

23. Prove that

$$-211 \cos 5\theta \sin 7\theta = \sin 12\theta - 2 \sin 10\theta - 4 \sin 8\theta + 10 \sin 6\theta + 5 \sin 4\theta - 20 \sin 2\theta$$

24. Find the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$.

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